AN IMPORTANT QUESTION

IN

METROLOGY,

BASED UPON

RECENT AND ORIGINAL DISCOVERIES:

A

CHALLENGE TO "THE METRIC SYSTEM,"

AND

AN EARNEST WORD WITH THE ENGLISH-SPEAKING PEOPLES
ON THEIR ANCIENT WEIGHTS AND MEASURES.

BY

CHARLES A. L. TOTTEN, M.A.,

FIRST LIEUTENANT FOURTH UNITED-STATES ARTILLERY; PROFESSOR OF MILITARY SCIENCE AND TACTICS AT "THE CATHEDRAL SCHOOL OF ST. PAUL;" LIFE-MEMBER OF INTERNATIONAL INSTITUTE FOR PRESERVING AND PERFECTING ANGLO-SAXON WEIGHTS AND MEASURES; AUTHOR OF "STRATEGOS, THE AMERICAN GAME OF WAR;" "OUR INHERITANCE IN THE GREAT SEAL OF THE UNITED STATES," ETC., ETC.

"Remove not the ancient landmarks which thy fathers have set."
PROVERBS xxii. 28.

NEW YORK:
JOHN WILEY & SONS,
15 ASTOR PLACE.
1884.

THE ARMS AND CREST

OF THE

UNITED STATES OF AMERICA,

AND THE

OBVERSE OF THE NATIONAL SEAL.

Ho! to the land *shadowing with wings*,—which is *beyond* the rivers of Ethiopia: that sendeth ambassadors *by sea*,—even in ships of *whirling things* upon the waters.—ISA. xviii. 1.

Franklin Press:
RAND, AVERY, AND COMPANY,
BOSTON.

TO THE
ENGLISH-SPEAKING PEOPLES
OF THE EARTH, WHO ALREADY POSSESS
THE WORLD'S GATES OF COMMERCE; WHO RAISE ITS
FOOD-SUPPLY, LOCK UP ITS SURPLUS, AND CONDUCT ITS TRADE;
WHO OWN ITS MINES, COIN ITS MONEY, AND CONTROL ITS INDUSTRY; WHO
INVENT ITS MEANS OF PROGRESS, CULTIVATE ITS INTELLECT, AND
ELEVATE ITS RELIGION; WHO TONE ITS MORALS,
LIBERATE ITS INHABITANTS, AND STEM
ITS TIDE OF ERRORS,
THIS BRIEF SURVEY OF A TOPIC PREGNANT
WITH WEIGHTY IMPORT
IS MOST RESPECTFULLY DEDICATED
BY THE AUTHOR.

"I swear unto the Soul of Isis, that I have not altered the Sacred Cubit of my fathers."

THE OATH OF THE EGYPTIAN DEAD (MONUMENTS OF EGYPT).

METROLOGY.

"But thou shalt have *a perfect and just* WEIGHT, *a perfect and just* MEASURE *shalt thou have*: that thy days may be lengthened in the land which the LORD thy God giveth thee." — DEUTERONOMY xxv. 15.

Ὁ Θεὸς μάλιστα πάντων γεωμετρεῖ.

"He created Wisdom, and saw her, and *numbered* her, and poured her out upon all his works."—ECCLESIASTICUS i. 9.

"Thou hast ordered all things in *measure* and *number* and *weight*."—WISDOM xi. 20.

PREFACE.

"And of Joseph he said, Blessed of the LORD be his land, for the precious things of heaven, for the dew, and for the deep that coucheth beneath,

And for the precious fruits brought forth by the sun, and for the precious things put forth by the moon,

And for the chief things of the ancient mountains, and for the precious things of the lasting hills,

And for the precious things of the earth and fulness thereof, and for the good will of him that dwelt in the bush: let the blessing come upon the head of Joseph, and upon the top of the head of him that was separated from his brethren.

His glory is like the firstling of his bullock, and his horns are like the horns of unicorns: with them he shall push the people together to the ends of the earth: and they are the ten thousands of Ephraim, and they are the thousands of Manasseh." — DEUT. xxxiii., 13-17.

THE race to whom these pages are dedicated is by blood, by letter, and by the spirit, literally "called in Isaac's name." As Saxons, they are the lineal descendants of the "Saka-i-Sunnia," or "Sons of Saac;" while, by the particular branch through which they derive their lofty genealogy, they are the posterity of Joseph, "the beloved son," "to whom pertained the birthright."

The Egyptian extraction of his two sons, Ephraim and Manasseh, bequeaths to them, together with all the blessings "of him who was separated from his brethren," an inheritance none the less royal and significant in the mysterious land of their mother, "Asenath, the daughter of Potiphera, Prince of On." Though, like another and a greater Son of Jacob, it is true that they were, in early days, "called out of Egypt," it is none the less true that the summons was simply given to them for the purpose of extending the possibilities of their ultimate dominion.

In these latter days, when the ends of the earth seem literally to draw nigh unto us, when every nation is so deeply impressed with the uncertainties surrounding the long-contested solution of "the Eastern question," when an intense and ever-increasing expectancy has settled down upon the whole human race, it may well be asked if it is not a little remarkable to see the flags of these two *brother* nations united for the first time, since their independence, in the streets of Alexandria? In 1882 the bronzed tars of England and America, of "Brothers John and Jonathan," — the only nations called "brethren" upon the face of the earth, — landed together on the shores of the delta of that ancient stream upon whose banks their fathers, also brothers, had lived as princes twice eighteen centuries before. Shall we endeavor to convince ourselves, that, in this act, there was no overruling exercise of that Will which weaves the thread of destiny? or shall we cease to doubt, and yield to the conviction that there is indeed a power that giveth the dominion unto whom it will?

In the same year, 1882, both England and America struck off commemorative medals, upon whose reverse faces the two most mysterious emblems of Egypt, the Sphinx and Pyramid, were severally displayed as central devices. In their inception these medals had not the remotest connection. The one was the Egyptian war-medal of Great Britain: the other was the centennial seal-medal of the United States. Nevertheless, a strenuous effort was made to induce the Queen to adopt the Pyramid as the central emblem for the British war-medal. It had already been used by America that very year; and it was not likely that an occasion for its simultaneous employment by the two nations would soon, if ever again, occur. But Ephraim is not Manasseh, and so the idea of the Sphinx was adhered to by our fraternal nation as for it the most appropriate. Was Providence, which counts the hairs upon a human head, also an unconcerned spectator then? And were these matters really trivial things, and, after all, of no historic moment?

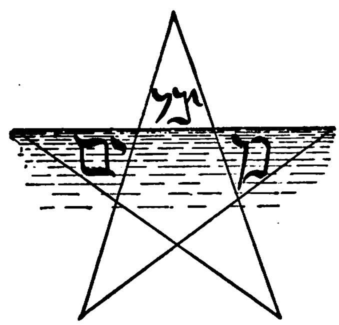
> "There's a divinity that shapes our ends,
> Rough-hew them how we will."

And it was no accident that the greatest commercial city of each of these two brother and Egyptian nations was at this same time graced with one of the two obelisks, that, when their father Joseph married the Princess Asenath, had stood, like Jachin and Boaz, in strength and beauty on each side the portal of her father's temple.

Who, indeed, shall say that, in youthful sports around the entrance to that noble shrine, their fathers did not choose, as children do to-day, and even name as "Ephraim" and "Manasseh," each one, the self-same pillar, which, in centuries then to come, the powers that overrule have now brought by such natural means to the more modern homes of their descendants, and have stationed at the very gates of all their greatness?

There is undoubtedly an inheritance in the land of Egypt for the Anglo-Saxon race, and the day has dawned when it shall be given unto those whose right it is.

The word צר (zr) is the Hebrew equivalent for the Egyptian hieroglyphic Ra, and this latter is one of the most significant names by which the Great Pyramid of Gizeh was known. It signifies *rock*, or *the rock.*



In the overflow of the Nile, this monument seems to rise out of the very water itself, and thus to be a unique symbol of the fabled land on which it has stood since long before the days of Joseph. The Hebrew word for *water* is מ־ים (maim); and if we read these two words, *water* and *rock*, combined, we have מצרים (mizraim), or the word employed for Egypt itself throughout the Hebrew text. It literally signifies *the rock out of the water.* Thus, as Mr. J. Ralston Skinner shows, the very name of this land contains a symbol of its importance in a picture which may be represented by sketching a river of *water* with a *rock pyramid* rising therefrom upon its bank. The word צר being put upon the pyramid, above the water, and the one מ־ים, so divided, being written beneath the water, a part upon either side of the pyramid. The two read, thus combined, from right to left, as in the Hebrew, give us as one word the famous name of Egypt, — the cabalistic symbol of the earth itself saved from the greater flood, and, even at an earlier date, drawn from the maternal waters of the very womb of chaos.

In the following pages I am going to request the descendants of these two branches of Joseph's family to look with me "unto the צר whence they are hewn," and listen to some of the momentous truths with which its metrologically proportioned blocks reply, in cosmic ratios, to the grand dimensions of the earth on which they live.

That Jehovah has said, "Out of מצרים have I called my son," is a fact significant of the essential importance which this land has always had, and ever will have, in the divine economy of the Scriptures, rightly understood. From the earliest record of the dawn of time, when first "the Spirit of God moved upon the face of the waters" (מים), unto its close, as described in Revelation, when, we are told, there shall be no more sea, the whole Bible is harmonious in its employment of these

roots; and whether it refers to the actual creation of the earth, the *macrocosm*, as in the first chapter of Genesis; or of the Pyramid, the *mesocosm*, or intermediate type thereof, as in the thirty-eighth chapter of Job; or to the lightening up of the darkness which covers the face of the deep submerging the unregenerated soul of man, the *microcosm*, as it does throughout its pages, — the same fundamental cabalistic play upon these mystic words occurs. Truly, this rising from the waters, this idea of *baptism*, has a meaning as far-reaching as the deep from which it springs. It is not in vain that creation and that the soul of man, covered with primeval waters, "cry out of the deep unto Him who hath formed the heavens and earth and all that therein is;" and we may rest assured, that in the beautiful proportions of that צר, thus raised from out the midst of the מים, at the centre and the border of this fabled land of mystery, — מצרים, — we may learn a lesson whose teachings to us, who in former days were, in Ephraim and Manasseh, thus just as truly lifted out of the Egyptian deep, will perhaps be of most momentous import.

In our treatment of this subject, we shall first examine into facts which lie near home, and see in how much, or in how little, the Anglo-Saxon race is actually in possession of the blessings promised unto Ephraim and Manasseh, as the sons of Joseph. We shall then ask our readers to accompany us through some studies of this remarkable monument, and by the way shall gather not a little from the God-designed metrology of Israel, of highly scientific import.

The hierarchy of science, so called, has long since agreed to disregard as totally unreliable every structure a single stone of whose foundation has been quarried from the eternal word of God. Any appeal to the Scriptures, no matter how modest, is so thoroughly at variance with the modern methods of philosophers, that the book of an author who has searched the Scriptures for guidance towards the eternal truth of things, is condemned unread, stamped with the seal of disapproval unopened, and burned relentlessly without a hearing, lest the people, having read it, should pronounce it true, and learn to disregard their would-be teachers.

But it is not to be expected that a book so full of stumbling-blocks, offensive to their theories of evolution, as is the Bible, should find any favor, or receive the least toleration, at the hands of modern scholars. From the dust they love to feel that all around has risen, and without a God; and into the silt of a disintegrated, dead, and formless universe, they trust at length their worn-out essences will sink into eternal sleep. Modern science, disguise it as we may, is thus at heart not merely far at sea upon the waves of doubt, but is essentially an atheistic school, that has no God, and which has long since closed its doors against the written Word.

From this school, therefore, the present volume does not expect a single meed of praise. But here I am content. Were it to be otherwise, — were recognition granted to the thoughts advanced, by our self-styled scientific teachers, — I should feel that the light in which these pages had been written was but an *ignis fatuus* from the swamp of things that soon must pass away. I write, however, for the people, whose concern alone this matter is, and who still read the Word with simple trust; and, though my subject is of the highest scientific nature, I doubt not they will find it clear,—as truth is ever found to be,—and not so intricate but that it will be filled with what to every mind is deeply interesting, and also find it to be well within the scope of even moderate education and capacity.

As a matter of fact, the Bible deals at great length with this very subject of Metrology. Around it the whole of the Hebrew polity harmoniously arranges itself. And very naturally too; for a just and perfect life was all it aimed at inculcating; and the very measure of fair dealing, of justice, and of truth, is centred in, and squared and righted at, an honest and an accurate standard, too sacred to be ever lengthened or diminished by any possibility of double dealing.

As the study of Metrology inevitably leads us to the study of the Great Pyramid of Egypt, so, too, it leads all dwellers in the land of Manasseh to look with greater interest on the arms and crest and seal chosen for the nation's blazonry by ancestors who wrought more wisely than they knew. The United States of America has been a nation marked out by special manifestations of Divine Providence from their very beginning until now.

It was in their earliest struggles that they looked towards this Western wilderness; and, behold, the glory of the Lord appeared in the cloud, and led them to their favored habitation. By faith, like Abraham, their ancestor, when called upon to go out into a place which they should afterwards receive for an inheritance, they obeyed, and went out, not knowing whither they went, and dwelt in their land of promise as in a strange country. But the clouds of the Almighty were about their habitation, so that the sun smote them not by day, neither the moon by night. It is, therefore, in their crest they fittingly commemorate how by faith there sprang from even one, and him as good as dead, so many as the stars of the sky in multitude. It is, therefore, in their motto they repeat this reference, and intensify the idea of union by that beautiful allusion to the universal brotherhood of all mankind, who in Christ, as Saint Paul truly says, are "*many in one.*"

This was, indeed, the nation that fled into the wilderness borne upon eagle's wings, — the Goddess of Liberty, clothed with the sun, bathed in the cloud-reflected colors of her flag, and crowned with the stars that marked the union of her States, and pointed out their lofty origin. And her eagle guardian was the Lord of hosts himself; for as an eagle stirreth up her nest, fluttereth over her young, spreadeth abroad her wings, taketh them, beareth them on her wings, so the Lord alone did lead his people towards the land of freedom, and there was no strange god with him.

Truly, then, may all the nations of the earth exclaim, "Who is like unto thee, O people saved by the Lord, who is the shield of thy help? thine enemies shall be subdued unto thee, and thou shalt tread upon their high places. In peace thou shalt be like unto thy father Joseph, a branch planted by the rivers of water that bringeth forth his fruit in his season: thy leaves also shall not wither, and behold whatsoever thou doest it shall prosper."

Then may Jeshurun, the wise people, fittingly reply, "There is, indeed, none like unto the God of Manasseh, who rideth upon the heavens in my help, and in his excellency on the sky. The eternal God is my refuge, and underneath are the everlasting arms."

Upon the reverse of our national seal, the references to our birthright as descendants of Joseph and Manasseh, and thus of Egyptian origin, are even still more pointed. The leading motto, — "ANNUIT COEPTIS," — "He has prospered our beginnings," is a direct use of an expression so often reiterated in the Bible-story of Joseph, that he has become the very type of "a prosperous man."

In the capstone we have again, not only the emblem of that Divine Providence which crowned our efforts as a struggling people, but of the Saviour of his people, in whom alone our building, fitly joined together, groweth upward into that perfect union of the human and divine. The building, — a pyramid unfinished, — an emblem of stability, of perfect measure, just weight, and of eternal truth, and harmony with nature, man, and God, is eminently the Egyptian emblem of Manasseh. The date upon its base is his year of maturity, — "MDCCLXXVI.," — and marks the dawn of another golden age, as the motto below expressly indicates: —

"NOVUS ORDO SECLORUM."

This motto is an intentionally altered quotation from Virgil's Fourth Eclogue, and was borrowed in turn by Virgil from the mystic Sibylline records. The text opens as follows: —

> "Ultima Cumæi venit jam carminis ætas;
> *Magnus* ab integro *sæclorum* nascitur *ordo*.
> Jam redit et Virgo, redeunt Saturnia regna;
> Jam nova progenies cœlo demittitur alto;
> Tu modo nascenti puero, quo ferrea primum
> Desinet ac toto surget gens aurea mundo,
> Casta, fave, Lucina," etc.

Translation.

> "The last age of Cumean song now comes;[1]
> *Novus ordo seclorum*[2] — a mighty order of ages is born anew.
> Both the prophetic virgin[3] and the Saturnian kingdoms[4] now return;
> Now a new progeny[5] is let down from the lofty heavens;[6]
> Favor, chaste Lucina, the boy[7] soon to be born,
> In whom the iron age[8] shall come to end,
> And the golden one[9] arise again in the whole earth," etc.

Words would be exhausted in any attempt to do justice to the thoughts that find birth in the contemplation of the American era. Unique with the rest of the symbolism upon the long-concealed face of our Great Seal, this motto comprehends in itself the whole of the Virgillic Sibylline fragment just translated. It is unique in its reference to the birth and genius of American institutions, — institutions that cannot pass away, and whose full development no hand can stay from reaching the goal of their most perfect realization. The new order of things has been let down from heaven, not again to be withdrawn from earth. But this motto is also most beautifully in harmony with the entire reverse of the seal itself; and it conceals a hidden reference to the Great Pyramid above, — the legacy of an *earlier* Golden Age to *ours*.

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[1] The seventh, or last, or perfect, cycle. [2] The sabbatic age of rest, — the age of freedom. [3] The Goddess of Liberty. [4] The new republic. [5] A progeny among nations, diverse from all predecessors — a government "of the people, by the people, and for the people!" — a nation of independent States, and yet the union of a multitude of individuals. [6] "The many and signal interpositions of Providence in our behalf." [7] "Young America." [8] Old-World ideas and bondage. [9] One founded upon individual liberty, freedom, and progress. "All men are born free and equal."

In this, "the last age of Cumean song," it is our task to rebuild the monument of just weights and perfect measures. In the day of liberty, now fully dawned, the recognized equality of all, demands, as the foundation of society, perfect justice in the dealings of man with man; and it is only in the rediscovery of the secrets of true pyramidal construction that the new order of the ages can be founded in stability.

When at length, therefore, we Americans, — as the children of Manasseh, — have fully come to read our title clear to this inheritance, so grand and so far-reaching, how pointedly will the blessing of the great ancestor who *adopted* us, — for we were *half Egyptian*, — and made us equal to his own, be *named* and *numbered* on us!

"*He*" (Manasseh, said Jacob when he blessed our fathers) "*also shall be a great people.*" Thus he, whose name was changed to Israel, made us greater than his own; since from them all he took the *birthright*, and conferred it upon the two adopted sons of Joseph! — upon Ephraim and upon Manasseh, upon England and America; *that is, upon the Anglo-Saxon race.*

CONTENTS.

PREFACE.

INTRODUCTION.

PART I.

WEIGHTS AND MEASURES.

PART II.

LINEAR MEASURE.

PART III.

CAPACITY MEASURE.

PART IV.

WEIGHT MEASURE.

PART V.

RECAPITULATION.

PART VI.

ÆROSTATICS.

PART VII.

CONCLUSION.

APPENDICES.

APPENDIX A.

APPENDIX B.

APPENDIX C.

ILLUSTRATIONS.

PLATES.

FIGURES.

METROLOGY.

INTRODUCTION.

"The powers not delegated to the United States by the Constitution, nor prohibited by it to the States, are reserved to the States respectively, or TO THE PEOPLE." — Art. X. Amendments to the Constitution of the United States of America.

METROLOGY is the science of round numbers, just measures, and of perfect weights. In it all other sciences find common ground. Its scope is co-extensive with the farthest reach of every special subject of investigation. There is no other subject that is worthy of sharing with it the holiest place in the temple of universal civilization. It is the very headstone in which the whole building fitly joined together shall find at last its own ideal fully realized. Upon the mount of intellect, and at the very dawn of time, its model was revealed to man; and ever since his aim has been to shape his mental architecture by it.

Metrology has been the question of the ages. It is the question of the present, and forever it will be a question of most vital import to the human race.

To a brief consideration of this pregnant subject, I invite the earnest attention of every statesman, and of every thoughtful citizen. To such of them as still revere the old traditions of our race, the review that we shall give cannot but prove both interesting and important; while to those who, willing to give up their birthright, are now so assiduously and insidiously advocating the repression of our hereditary weights and measures, and the compulsory adoption of a foreign system with which our practice and our history have naught in common, the gauntlet is thrown down.

We challenge them to an open and a free discussion. Let them produce their case. *It is the* PEOPLE'S QUESTION, — *this one of* WEIGHTS AND MEASURES; *and as a right inherent, and one never ceded in the* CONSTITUTION, *they alone can change the "times and seasons."*

Our representatives have no more right to force the metric system upon us than they have to make our babies beg for bread in foreign idioms. It will be objected by some, that §5 Sec. viii., Constitution of the United States, conveys to Congress the direct power to "fix the standard of weights and measures." This is the exact wording of the document, but it by no means implies all that the advocates of change would have us believe. A *standard* is one thing; the *unit* quite another: thus Congress has already and legitimately made the *yard* the national standard of linear measure; but as to the *unit*, that is fixed. The inch has never been disturbed, nor does the right inherent exist in Congress to disturb it. It was never contemplated that our representatives would sweep away our units, any more than that they would attack the roots of our language. Moreover, the very context of this paragraph convinces us that the province of Congress was merely to regulate and define the standard in such exact terms, that, as in the case of the value of foreign coins, etc., future legal complications as to values in the interchange of commodities might be avoided. Within the limits of the Anglo-Saxon system the power of Congress is clearly confined; and the minute it passes to *new units*, or beyond the borders of our native metrology, expressed and handed down from the remotest times in Anglo-Saxon terms, and linked to Anglo-Saxon history, — at that minute it trespasses upon rights undoubtedly reserved unto the people. If the French system of metrology is to become Anglo-Saxon, it must become so by the popular vote of those whose right alone it is to assume the inconvenience and the great responsibility.

Let those, then, who are so continuously knocking at the doors of Congress come rather to the forum; let them mount the rostrum in the market-place, and show their wares to those who buy and sell. It is there alone the question can be settled. A law made elsewhere upon such a subject can be enacted but in vain.

In the mean time we offer to the people the following brief review of our *native* Anglo-Saxon metrology, and claim that it is the most truly earth-commensuric system now in use among men. It needs but a slight *rectification* to make it absolutely perfect.

This "rectified system" is our special theme. As an associate of the International Institute for Preserving and Perfecting (Anglo-Saxon) Weights and Measures, these pages originated in an address intended primarily for them. But from the important character of the discoveries to which our researches have led us, we are now induced to offer their results to the Institute in a more public manner, and to invite all others who are interested to an open meeting and a free discussion.

Human law is absolutely powerless to *enforce*, as the UNIT of metrology, that which is not harmonious to nature. Our venerated and beloved GARFIELD,[1] in a speech at Boston in 1878, regarding the *standard of weights and measures*, said, —

> "I challenge the intelligence of any man who hears me, to think of such a thing as a *measure* of *length* which has no length in itself. No: by laws HIGHER *than human legislation, length, depth, and height* were created; men can only name and declare a definite length as the standard."

We believe that our ancestors have bequeathed to us a system based upon these HIGHER laws. By the attrition of full four thousand years, our Anglo-Saxon system has lost so little of its ancient truth, that we believe its present possessors — direct descendants of a mighty race — *may return to the ancient* PERFECT *standards without any inconvenience, and without altering a single name familiar to our children and our history.*

To a survey of these facts we therefore invite that race, which hitherto has not removed its ancient landmarks, and ask

[1] Garfield was the first elected president of this Institute. He took a deep interest in the cause. In his letter of declension, Nov. 29, 1879, he says he thinks he can serve the cause more effectually, and without indelicacy, as an independent judge, when Congress, of which he is a part, shall be memorialized in relation to measures the Institute may bring before it.

that every Anglo-Saxon give them all the consideration that they justly merit. The subject is too important, the time too critical, the interests involved are too far-reaching, and *the labors of our national adversaries are too incessant*, to admit of any loss of time. We bring this matter to the bar of the people themselves; and to that bar we bid our adversaries come, and bring this challenge with them.

In the following pages, the present Anglo-Saxon linear inch being taken as unity (1.), an "earth-commensuric," or "pyramid-linear" inch will be expressed by 1.001, ± an infinitesimal correction yet to be determined by astronomers. In the text, it is sometimes referred to as a rectified inch, and at others as a *pyramid* inch. Those familiar with the pyramid literature of the present day will of course understand these distinctions without the foregoing explanation. The controversy now going on among scientific men, relative to the true import of the Great Pyramid, is waxing hotter every day, and at the same time is becoming more dignified. Even the opponents of what is opprobriously termed the "religious theory" by Mr. Proctor, are becoming more and more convinced that the mystery of this ancient mountain of the Nile is not yet solved, and that the secret of the greatest of the world's seven wonders, — the only one yet standing on the earth, — has not been watched without a purpose for so many eras by the silent Sphinx. To both sides of this controversy the following pages will have much to say. And to those without the arena, the merely interested laity, the whole topic, *developed now upon most interesting and important lines, assumes the proportions of a world-wide question.* No subject that has ever engaged the attention of the intellect has elicited the interest that now surrounds the solution of the problem of the Great Pyramid. It has held the attention of man in every age, and to-day retains it more intent than ever. If any fair-minded, candid spirit once enters upon the subject, it will never cease to be absorbing; and the grand truths of nature that are now unfolding beneath the general scrutiny focused on it, will be seen to rise pyramidally far above the loftiest subjects that have ever held the mind entranced. Nor should the moderately informed turn hopelessly aside, for fear the theme demands more erudition than is generally possessed. The subject is as simple as the figure of the monument itself; while at the same time, in its scope, it is so mighty that the loftiest genius may find its measure far beneath its dizzy summit. Its story may be made a nursery-tale, as easily as it becomes a midnight vision in the study of the great philosopher. The only answer, then, to him who asks, "Can any good come out of Nazareth?" is that made years ago, and still as full of meaning, — "Come and see."

I.

WEIGHTS AND MEASURES.

"Just measure and a perfect weight,
Called by their ancient names."

"The Philistines be upon thee, Samson." — JUDGES xvi. 9, 14, 20.

Why Anglo-Saxon Metrology should not be abandoned. — The Metric System *versus* the Anglo-Saxon. — The English-speaking Nations, and the Commerce of the Earth. — The Balance-sheet of the World. — Remarkable Facts, and the Future of Anglo-Saxondom. — Our Duty certainly to preserve and perfect our own System. — Destroy not the Ancient Landmarks.

HARDLY a year passes that either openly, or in covert ways, the National Legislature at Washington is not asked to pass laws or resolutions looking towards the ultimate adoption of the metric system of France.

Fortunately, however, Congress is slow to move in such matters, and has hitherto shown a disposition to regard this subject rather as a national one, and therefore as one for "the people," as such, to settle at the polls.

The design of bills such as that of the late Alexander H. Stephens, is to make *compulsory* the use of the new French device in place of our present system of weights and measures, regardless of our great pecuniary loss, and the lamentable evils it would necessarily entail upon our people. Thanks to the efforts of the Institute, to which this paper is more particularly addressed, such bills have hitherto come short of their contemplated ends. It is a source of regret, however, that this foreign system has received at the hands of the authorities at Washington even a *permissive* sanction. There was no need of any such act of recognition. In a free country the metric system has equal rights with any other, and should stand upon its own merits. It should certainly have received the tacit adherence of the majority, before any such public step or act bringing it into notoriety was advisable. The advocates of this system, however, are too deeply pledged to the ends they have in view not to take advantage of the natural apathy of a people merged in business and industry of every nature; and so, without our knowledge even, we find them strenuously at work in every direction, introducing it into all our schools and colleges, and here and there, without even the show of legal right, forcing it into the very acts of the government itself.

"Every man of genuine *practical* experience realizes the absurdity and impracticability of substituting this inconvenient French metric system for our own hereditary system of weights and measures, which has been in use from time immemorial, and has woven itself into all our history.

"Now the great danger lies in the encouragement given to the persistent advocates of the metric system by those who are *indifferent*, *unadvised*, or *ignorant* of its demerits in comparison with the merits of the system to which it is opposed."

The arguments of the metric philosophers are mostly specious ones, and will not bear a deep research into the constituted nature of things. Its decimal feature is the only one that demands serious consideration; and this is but a borrowed one, — by no means French, — and one of doubtful importance to man, who habitually quarters and halves, and in all matters of dimension and metrology seems to comprehend *common* fractional ideas more instinctively than he does those of tens. Indeed, the very continuity of the French system itself has been broken by the introduction and constant use of halves and quarters throughout its series. Such natural subdivisions have been found as absolutely necessary as the half and quarter dollar is in our own decimal system of currency.

In a country constituted like ours, the subject of so radical a change as would be involved in an entire removal of our ancient metrology is certainly one for the popular ballot only to decide, and one with which, when its true import is made manifest, even politicians (to say nothing of *statesmen*) will be found loath to identify themselves.

The time has therefore come when it is necessary that the people should be informed what this danger means. It is a

"Battle of the Standards." It is Anglo-Saxon metrology *versus* the Metric System. The one or the other must become obsolete, and that right speedily; for we are now fully within the threshold of an age that demands the recognition of universal ideas and standards over the whole surface of the globe. The easy introduction of standard time into this country, the unanimity with which the meridian of Greenwich has been accepted as the standard meridian for geographical and commercial purposes, are significant. The next step is naturally one adoptive of an universal system of weights and measures. Some twenty nations have already recognized that of France; and in name, at least, the metric system is already international.

But it is not so in fact. It is in the Anglo-Saxon system that the actual business of the world is even now transacted. This is a startling statement, for it puts an entirely new phase upon the question in controversy. It is a novel and interesting stand-point, from which, as Anglo-Saxons, we can afford to review the whole topic before finally committing ourselves to an almost irrevocable decision.

From such a stand-point the subject can be dealt with as it were *ad hominem;* and from it we will be surprised at the overwhelming facts which go to establish beyond a doubt that the Anglo-Saxon system is, in reality, the *de facto* "International system" of the earth to-day, and that the irresistible march of events has already given it, and will continue to maintain it in, the ascendency, until all rival systems, particularly the metric, shall have passed quietly into oblivion.

If any one is at all doubtful of the future of Anglo-Saxondom, or disposed to question the statistical FACT that already, to-day, this race stands head and shoulders above all other peoples of the earth, and is moving onwards, in every line of progress, at a constantly more accelerated rate than they, we refer them to a volume which should be in the hands of every English-speaking statesman, — "The Balance-sheet of the World," by M. G. Mulhall, F.S.S., London.

As generic, and essentially preliminary to our subject, we propose here to briefly review some of the startling facts presented in this work. By such an examination it can and will be conclusively shown that Anglo-Saxon metrology is by no means the incongruous failure its adversaries have endeavored to demonstrate, and to make it clear, that, if without any particular attention it has already accomplished so much, it will certainly accomplish all metrologists can wish for when once it shall have been unified and rectified.

In a paper upon "Weights and Measures," written in favor of the metric system, and delivered before the American Society of Civil Engineers, Mr. Frederick Brooks, C. E., laid great stress upon the fact, that, of the imports into the United States (which, in the year ending June 30, 1879, amounted to $445,-777,777), only twenty-eight per cent was produced in Great Britain and countries using the Anglo-Saxon weights and measures; while more than half (fifty-nine per cent) was produced in countries, that, within the last hundred years, have adopted a common international system of metrology. The remaining thirteen per cent was produced in countries which use various other systems of weights and measures. These facts were all exhibited in detail by Mr. Brooks, in an accompanying diagram; and from them, as of primary importance, it seems as though we were expected to draw the conclusion that Anglo-Saxons, at any rate those of the United States, should also unhesitatingly come into the International Congress, and speedily adopt the metric system.

At first glance this inference seems to be of some weight, but it is only so upon its surface. Indeed, the reverse consideration of the subject entirely overturns it. The argument is drawn entirely from our imports, which are sold to us, and therefore from things in which naturally the purchaser is enough concerned to look after his own interests, and learn the metric system, if needs be, to better guard them. But importing is by no means the business of this country. It certainly did not monopolize our wealth in 1879, and still less does it do so to-day.

The combined industry of the country in 1879 was represented by a money (gold) value of some two thousand million pounds sterling, or at least ten thousand million dollars, of which the import business barely represented twenty-two per cent.

Shall the seventy-eight per cent involved in other industry adopt "new times and seasons," and change its manners and its customs simply to accommodate the business of so small a minority? Surely one must have a strange idea of American institutions, to found an argument upon such a basis, and a stranger one of American perspicuity, if it is presumed that an audience will draw such conclusions even from such data, to say nothing of those which result from a more general consideration of the whole subject.

Now, the imports into the United States are generally luxuries and high-priced articles, or only raw material. They are mostly the surplus from arts and trades of long standing in foreign countries, and from them in particular is derived the wealth of the countries whence we procure them. In times of patriotic war, our people have given ample proof of their willingness to sacrifice them all.

But there is another and far more important side of the subject, — that of our *Exports*, — with which foreign countries are now so deeply concerned.

In the very year selected by Mr. Brooks for this discussion, our exports were in value $736,634,834, or in excess of our imports by $290,000,000. This excess alone is more than half of the total imports for that year. Moreover, if there be any value in the implied argument of Mr. Brooks, it is, of course, turned back upon his own conclusions from this opposite stand-point. Of our exports, $426,000,000, or about sixty-six per cent, went to the Empire of Great Britain alone (an empire using Anglo-Saxon metrology); *and this amount, let it be noticed, was practically equal to the bulk of our whole import business.*

With our exports it is not as with our imports. While we can easily get along without the latter, the world itself cannot live without the former. We hold the food-surplus of the earth; and the bulk of our exports is in breadstuffs, provisions, meat, etc.; that is, in staple articles. Our exports in food, in 1879, were thirty-two times our imports ($10,000,000) therein.

While carefully examining this matter, it is intended that the data used shall be drawn largely from the London book already referred to. Speaking of the United States, the author remarks, "Ten years ago the balance of trade was against the country, but now the exports are thirty-one per cent over the imports." This is still more true in 1883. "The Americans now make one-fifth the iron, and one-quarter of the steel, of the world. . . . The United States raise one-half the gold, and one-half the silver, of the world's supply. . . . Taking *in globo* all the mining-industries of the world, the United States represent thirty-six, Great Britain thirty-three, and all other nations thirty-one, per cent of the total." Thus Anglo-Saxondom represents sixty-nine per cent of the mining-industry of the earth.

"The sailing-vessels of the world now trade mostly to the United States." This being a fact, it follows, that, no matter what weights and measures they use at home, they use the Anglo-Saxon ones in our own markets. "But in comparison with commerce, the Americans use three times as much money as the English, and nearly twice as much as all Europe." Moreover, in the past ten years the United States has coined one-fourth of the gold, and one-sixth of the silver, used by all the earth. "The net income of the United States per inhabitant is double the European average."

America is a peaceful country: its ægis holds the olive-branch of tranquillity within its hand of friendship. Our military expenditure is the least of all nations, and is less than one-fourth of the general European average.

Our national debt has been reduced twenty-two per cent in ten years; in fact, the ratio of debt per inhabitant has fallen forty-two per cent, that of interest fifty-four per cent, in ten years. "Population has increased thirty-one per cent since 1870 (i.e., to 1880), being the largest number gained in any decade of the Union." The increase of births over deaths "is three times the average European increase, and double that of England or Germany." It is even superior to that of Australia, a newer country. "Every thing seems to promise, that twenty years hence, at the close of the nineteenth century, the United States will have between ninety and a hundred million inhabitants."

"Food supply is so abundant that the grain-crop is eighteen and a half per cent over consumption, and the meat thirty-six per cent in excess. The United States produced thirty per cent of the grain, and thirty per cent of the meat, of the world."

"It appears, that, in spite of the population increasing 1,250,-000 yearly, the supply of grain is growing faster, and of meat as fast. So that the exportation to Europe is likely to go on rising for many years to come." In spite of the home consumption of meat being 120 pounds per inhabitant, equal to 2,740,000 tons, a surplus of 1,076,000 tons is annually left, one-half of which is exported to the over-populated and hungry foreign nations. "The Americans are apparently the best fed of all nations." They are likewise the most comfortably clothed in cottons, woollens, and linen.

Concerning our railroads, telegraphs, and internal systems of river communication, it is needless here to speak: they have no compeers on the globe.

Now, in view of all this wealth, and the constantly increasing importance of this country as the market of the whole world, how ridiculous is any appeal to our imports as an argument in favor of changing our hereditary weights and measures, and abandoning, to our inconceivable discomfort, our own traditions for those of other nations, or for traditions which are no traditions, — as those of France are not a century old! No: rather let all other nations who buy, who are glad to buy, and who *must* buy in our markets double, ay, quadruple, what we buy abroad, *return* to their own "ancient landmarks;" and, in the mean time, let us only strive to *perfect* the heirloom we have hitherto so well preserved.

Thus far this argument has only been answered from a single stand-point, — that of our own nation, — and it should be borne in mind that the statements have been drawn from the candid pages of a foreign book. There is a grander view to take of this topic. It is from the stand-point of all Anglo-Saxondom compared with the rest of the world at large. The world has increased in population in the decade from 1870 to 1880 about 9¾ per cent. In the mean time, Great Britain increased 10½, the United States 31, Australia 56½, Canada 14¼, and South Africa (Anglo Saxon) 73¼ per cent. No other nation, save Belgium, has increased over 11 per cent, and France but 1.67 per cent. Even Turkey (2.01 per cent) has increased at a higher rate than the mother of the metric system. At such a rate of increase all other nations must in time be smothered out, and France among the very first, before the Anglo-Saxon race.

But, further, the tabulated statement of the port-entries of all nations for the ten years under consideration shows, that, while the tons burden of the world were 50,000,000, the United Kingdom, British Colonies, and the United States contributed 28,000,000, or more than half.

One-half of the whole industry of the world is already in Anglo-Saxon hands. In millions sterling the increase for the ten years was, for all the earth, 1,866; while it was for Great Britain 337, United States 525, Australia 57, Canada 28, and South Africa 14; making a total of 961 millions of pounds sterling *increase.* To this increase can also fairly be added that of South American industry, 24 millions, almost all of which is represented by British capital. In 1880 the industry of the earth, expressed in millions of pounds sterling, was 2,024 for Great Britain, and 2,004 for the United States. These two nations headed the list, being followed by France at 1,325, by Germany at 1,269, and by other nations at a greater distance. By industry we mean commerce, manufactures, mining, agriculture, carrying-trade, and banking.

The increased consumption of cotton, wool, flax, jute, etc., in the decade has been £1,666,000,000, of which £922,000,000 — much more than half — has been in Anglo-Saxondom. The increase for the world in manufacturing has been £558,000,000, of which £324,000,000 has been among those using pints, and pounds, and inches. That for all Europe (non-Anglo-Saxon) was but £212,000,000.

Out of 118,000,000 tons increase in production of coal, Anglo-Saxon weights have measured 78,000,000. Out of 7,233 increase in thousand tons of iron, they have measured 5,250; and of steel, out of 3,068, they have measured 2,255; i.e., in every case far more than half of all the earth. Anglo-Saxondom produces and measures out by the pound and yard more than fifty per cent of all the wool, and the United States alone seventy-five per cent of all the cotton, raised upon the earth; and other nations are glad to purchase all these things in pounds and yards. In general terms, the study of the commerce of the world for the past twenty years (1861 to 1880) shows, that, out of £40,000,000,000 (giving the value of the exports and imports in round numbers), Anglo-Saxon metrology has measured and *re-measured* far more than half.

The value of the shipping of the earth has increased

£40,000,000 in the decade, £26,000,000 of which was in Anglo-Saxon bottoms; that of all the metric nations put together was only £13,000,000, the remaining £1,000,000 being scattering. *Thus two-thirds of the carrying-trade is already Anglo-Saxon, and but one-third "metric."* Which, therefore, it may well be asked, is the *de facto* international system? Does it not rather appear that France, leading the Opposition in "the Napoleonic day," when all her interests are known to have clashed with those of England, strove for the mastery in commerce by this politic though vain attempt at banding subjected Europe in a new metrology? And does not the irresistible march of industry prove that her dying system is international only in its self-assumed, high-sounding name?

Again: in 1879 the "tonnage" on sea of the earth was 18,000,000, 10,000,000 of which was Anglo-Saxon; the "carrying power" on sea was 34,000,000, 21,500,000 of which was Anglo-Saxon. For the United States alone the carrying-power at home and on sea was 9,000,000 tons, and the tonnage 4,500,000.

There were 882,000,000 passengers carried upon Anglo-Saxon railways in 1879, against 1,497,000,000 for the whole world, and against 603,000,000 for the whole of Europe, — "the Continent," — only partly "metric" after all, since the great Russian Empire still remains without the metric union.

There were in 1879 in England 26,000 miles of active telegraph-wire, in the United States 119,000, and in the British colonies 59,000; that is, for all Anglo-Saxondom 203,000, against 250,000 for the partly metric Continent, and against but 303,000 for all the rest of the world. But upon these wires, as an element going to show the magnitude of their thrift, business, and enterprise, the Anglo-Saxons sent twice as many messages per inhabitant as the Continental nations. England sent 77, the United States 62, and the Continent but 30 messages, per hundred inhabitants.

Since 1870 (and to 1880) the mines of the earth have produced £360,000,000, of which £215,800,000 were from Anglo-Saxon mines. And Anglo-Saxon mints have coined in the same decade £224,420,000, out of £526,781,000 coined over all the earth. Furthermore, out of £905,000,000 in coined specie current in the decade ending 1880, £524,000,000 were used in Anglo-Saxon import business, against £367,000,000 on the European Continent, and £514,000,000 in export business, against £339,000,000 upon the Continent.

In accumulated wealth in 1880, Great Britain and the United States led all the earth, followed next by France and Germany, and far behind by all other nations taken individually.

The accumulated wealth of the whole European Continent was £28,000,000,000, that of the Anglo-Saxon nations £18,000,000,000, while that of the world was only £47,000,000,000. Considered from another stand-point, the world had £113 per inhabitant; Europe, including Great Britain, £111; Great Britain alone, £260 (more than any other nation except Holland (£283), and more than double that of the world); the United States, £158; Australia, £172; and Canada, £148. Thus the Anglo-Saxons, as individuals, are worth per inhabitant some £184; the world average being £113, and that of the whole continent of Europe being but £91.

But a consideration of the public debt of nations is even more significant. In the decade 1870 to 1880 the debt of all Europe increased £52,000,000, and that of the world £44,000,000. But three nations of the earth effected any reduction of their national debts. These were the United States, by £86,000,000; Great Britain, by £24,000,000; and Denmark (a former "resting-place" of the Anglo-Saxon), by £3,000,000. The public debt of the world in 1880 was £5,207,000,000, that of the continent of Europe alone being £4,513,000,000; while that of all Anglo-Saxondom together was but £1,276,000,000. Now, it is also noticeable in this connection, that, while the debt of Anglo-Saxondom is almost entirely held in native hands, that of the rest of the world is in foreign hands, and that far more than one-half of it is actually held by Anglo-Saxons. This race, in fact, has bonds and mortgages on all the world.

In earnings the United States lead all other nations, their earnings for 1880 being £1,406,000,000. They were followed by Great Britain and her colonies at £1,381,000,000, and far behind by France at £927,000,000, by Germany £851,000,000, and by other nations at continually lower figures. The earnings of the Continent were £3,797,000,000; of the world,

£6,773,000,000; those of Anglo-Saxondom being £2,787,000,000, or more than two-thirds that of the Continental nations, and far more than one-third of all the earth.

Furthermore, taxation has declined in Anglo-Saxondom alone. In Great Britain it has declined in its ratio to income, while in the United States it has done so, not only notably in this relation, but also in the absolute. In every other nation taxation has increased both positively and relatively.

As to the food-supply of all nations, Europe in 1880 had a deficit of 380,000,000 bushels of grain, while the United States alone had a surplus of 370,000,000 bushels. In tons of meat, Europe had a deficit of 853,000, while Australia alone had a surplus of 838,000, the United States of 1,076,000, and Canada of 170,000. Of the grain-surplus of the world (22,000,000 bushels), in 1880 17,000,000 were held by Anglo-Saxons; and of the 2,144,000 tons of meat, then surplus, 1,931,000 were also owned by Americans, Australians, and Canadians. The balance was held in South America and Algeria, and almost entirely controlled, as in fact is almost all South-American industry, by English capital.[1]

But to what purpose shall we here continue this interesting review of man's affairs? The world is already Anglo-Saxon; and, in the face of such figures as we have just reviewed, the claims of those who continue to urge us to adopt the metric system are merely vain words, and uttered to no purpose.

The figures we have quoted are *facts*, — hard facts, — and from their very nature they are international facts. They show that the metric system is only international in name, and that *the truly international system of metrology is, in fact, our own Anglo-Saxon one.* To abandon it to-day, when all the world is really using it in buying food and raiment at our ports, would be to introduce more confusion into human affairs than the earth has seen since the days of Nimrod. In changing Anglo-Saxon weights and measures for the metric system, we would

[1] For further sources whence to draw even more significant deductions as to the signs of the present and the future of the Anglo-Saxon race, the reader is referred to The Balance-sheet of the World, The Progress of the World, The English in South America, Hand book of the River Plate, Hand-book to Brazil, etc., all by Mitchel G. Mulhall, F.S.S., London, 1881.

not only disastrously and to no purpose disturb our own affairs, but inconvenience those of all mankind. What the English-speaking races, therefore, need to do is, not to adopt an alien system, but to *perfect* their own.

"DESTROY NOT THE ANCIENT LANDMARKS."

Rather let us strive to unify the system, and to rectify it back to its original and grandly earth-commensuric proportions; then may we transmit it proudly to posterity, an heirloom still more valuable than when it was intrusted by our sires to us. Let us, therefore, continue our investigation of this *de facto* international system of metrology, — that of the Anglo-Saxon world, — and determine how far it is from being perfect, — from being actually earth-commensuric, — and thus in how much it must be improved and unified and rectified, in order to make it more than ever, and for all future time, a blessing to ourselves and to our fellow-men.

II.

LINEAR MEASURE.

"So long as the human mind remains mathematical, it will prefer a diameter to a circumference." — HERSCHEL.

Our Linear Unit, the Inch, grandly Earth-commensuric. — An Additional Standard Table proposed for Decimal and International Purposes. — The Possible System absolutely perfect. — Our Present Tables may be rectified, and retained so long as useful. — No Change of Names necessary.

IF all human life were at this moment to be blotted out, and every book and instrument destroyed; and if this building only in which we sit to-night were left standing upon the surface of the earth; and if a body of scientific explorers, coming from some other planet, were to be landed here, — do you think they could rediscover the unit of length used in its construction? Undoubtedly they could.

No matter how different their own linear unit and standard might be, they would find, by its successive applications to the various parts around them, certain repetitions, doublings, divisions, etc., and could not but at last arrive at the conviction that an inch had been the unit, and a measure of twelve such inches the standard, of its builders. They would not of course call it a "foot," nor know the unit as an "inch;" but they would not fail to conclude that these two lengths were the common divisors of all the measures they could make. They would soon be positive in their conviction, and assert it as a fact. Nay, further: if they were equal to ourselves in science and astronomy, they might ask themselves why we had employed the smaller one, the inch, as a unit; and perhaps in their natural study of the earth itself, and in the light of intellect (which must be similar throughout the universe), they would at last discover why, in the fact that an "inch" was exactly commensurable with the grandest dimension of the earth itself.

This is just such another investigation, as, during the past fifty years, has been brought to bear upon the ancient pyramid; and we are now as sure that it was constructed with a unit inch, and by a standard cubit of twenty-five such inches, as if its stones should cry aloud, and tell us its dimensions. We also know that this inch is the most perfectly earth-adapted unit of which we can conceive; and all this information, and whole volumes of astronomic and other metrological formulas, have in the mean time also been discovered, as due solely to the architectural study of this ancient monument.

How beautiful a conception is that of subdividing the polar radius to obtain a standard! The axis of the earth is its shortest diameter; about it the habitations of man are all conveniently disposed. It is a line so grandly fixed, that, through all the cycles of time, it knows no shadow of a change in length or in its general direction. About it day and night unerringly return. It sweeps the sky in a majestic cycle, that but once in 25,827 years returns into itself, and forms a dial upon which the duration of races, and the eras of geology, may well be measured. One-half of its length, the polar radius, is the very unit which astronomers employ in measuring distances within the solar system.

This terrestrial polar radius forms in turn the nobler unit of a still greater radius,—that of the solar system itself,—with which ambitious man essays even to pass beyond, to stars and systems floating upon the very horizon of siderial immensity. Who, then, can fail to see the grandeur of such a unit of metrology, or fail to catch the beauty of a system of such universal application?

Without at present reviewing the pyramid system of linear measure, as heretofore tabulated by Professor Piazzi Smyth, let us proceed directly to the consideration of our Anglo-Saxon linear measures, and try and determine how they may be rectified to earth-commensuric utility, duly prepared for decimal use; and, while being thus *perfected*, be at the same time jealously *preserved* in all those essentials of name, familiar *rationale*, and common-fractional sequence, which tradition, time without beginning, has made so sacred to us as a race.

Our Institute has been frequently accused of giving up too much time to pyramid studies, and of leaving out of consideration the more important subject of the improvement of our tables. I therefore propose to consider the subject of Saxon length-measure simply and solely from the stand-point of earth-commensurability, and fitness for future international use.

If some of us have already found the ancient monument of Egypt to teach the same great lessons, and if the world at length shall read the "stone book" in the same bright light, then, as pyramid-students, we shall of course rejoice; but, as an institute, we are incorporated to *preserve* and *perfect* the Anglo-Saxon system as our prominent object.

It is of little consequence, then, where we find the truth, so long as what we bring up is the priceless pearl itself, and particularly so long as we can prove its value from its own inherent radiance.

Without further circumlocution, therefore, I shall ask your attention to what, it seems to me, offers a practical means of accomplishing the ends for which, as a society, we exist.

Years ago the great Herschel advocated the measurement of the polar radius of the earth, the adoption of its one 10-millionth as the standard international measure, and that a twenty-fifth ($\frac{1}{25}$) thereof be taken as the primal unit of all measure whatsoever.

Time has shown the wisdom of his proposition. And now at length, in our day, we find the very president of the metric bureau in this country — President Barnard of Columbia College — regretting that the radius was not taken as the basis of the French metric system; and publicly declaring, that, were the metric system to be formed anew, it would undoubtedly be founded on this very line, so sure and fixed. Not only, therefore, impressed with the singular fitness of such a measurement for standard and unit purposes, but profiting by fair experience, and by the candid admissions of those who have already given up their own traditions, let us cling to ours, and endeavor to right them at this, the noblest dimension of the earth on which we live.

For the unit of linear measure I therefore join in proposing that this Institute resolve upon and adopt the exact five-hundred millionth of the terrestrial axis; and that this unit be named after its present wonderful Anglo-Saxon approximation, and called "one inch."

The value of the unit will not differ from that of the present Anglo-Saxon unit inch (British or American) by more than .001. Of this we are *already certain* from our present knowledge of earth size and shape.

In order to obtain this measure exactly, and for all future time; to provide for its preservation and accurate practical renewal at any time; and to establish it beyond danger of loss or variation, — I also propose that the Institute shall offer a prize for the best essay upon the following theme: —

"What is the simplest and most certain method of practically obtaining and preserving the measurement of the polar axis of the earth?"

The method proposed should be based upon the universal principles of sound astronomy, be convenient of application, avoid national prejudice, be accompanied by a full and complete mathematical demonstration of its certain accuracy, and be insured, by several checks, against the internal mechanical errors of calculation.

The inch is the grand central unit of the entire Anglo-Saxon system; and its numerical and earth-commensuric value must be rigidly determined before we can reasonably attempt to perfect any other part of the wonderful inheritance we have received from earlier ages.

It should be clearly manifest that we must needs have actually in our possession the measured length of this unit before we can determine the numerical nature and earth-commensuric relations of any others; as, for instance, those of capacity, weight, area, volume, etc.

We may, however, in the mean time, profitably study and examine the internal arrangement of the various subordinate branches of the Anglo-Saxon system of metrology, and determine what ought to be, and what shall be, the future and international arrangement of the whole system.

It is an entirely feasible proposition to determine the *rationale* of a system before we have obtained (or fixed by resolution) and adopted the exact *numerical* length of its desired units; thus, for instance, the foot, the yard, the 24″ gauge, — long measure, in fact, should be carefully maintained and preserved. It is of manifest utility, of traditional value, and will find employment to the end of time. In the mean while it will probably be necessary to adopt some *decimal* system founded upon the inch, for purposes of rapid calculation and international use.

Without reference, therefore, to the *absolute* length of the unit inch, save that it shall be, when determined, accurately one 500-millionth of the polar axis (and until so determined shall be held at its present statute value), we may agree beforehand that the *rectified* long measure shall be as heretofore, and as follows: —

RECTIFIED LONG MEASURE (STATUTE).

1 inch = the central "*unit*" = one 500-millionth *polar axis;* may be subdivided, as at present, either decimally or common-fractionally, to suit necessities.

$\frac{1}{72}$ inch	= 1 point.		
$\frac{1}{48}$ inch	= 1 hairbreadth.		
$\frac{1}{12}$ inch	= 1 line.		
$\frac{1}{3}$ inch	= 1 barleycorn.		
3 inches	= 1 palm.		
4 inches	= 1 hand.		
5 inches	= 1 span.		
10 inches	= 1 double span.		
12 inches	= 1 foot.		
2 feet	= 1 gauge (24-inch).		
3 feet	= 1 yard	= 36 inches.	
6 feet	= 1 fathom	= 72 inches.	
$5\frac{1}{2}$ yards	= 1 rod	= $16\frac{1}{2}$ feet	= 198 inches.
40 rods	= 1 statute furlong	= etc.	
8 furlongs	= 1 statute or land mile	= etc.	
3 miles	= 1 statute or land league.		

Let a "cable's length," as heretofore, equal 120 fathoms or 720 feet; and let "Gunter's chain," "Superficial measure," "Cubical measure," etc., remain upon our books for those who desire to use them. So long as they are valuable, and subserve a purpose, they will be used: when they cease to be so, they will naturally fall into disuse.

For *decimal* purposes I propose the following system, based upon pyramidal studies, and upon the traditional lore of our race; and to distinguish it from the present statute or long measure, I shall denominate it "Standard Linear Measure." When I come to the subject of capacity and weight measures, as *rectified* by certain principles which I have discovered to exist in the constitution of all nature, the parallelism of the following table will be more clearly manifest. Its facility for mathematical use is of course already apparent.

STANDARD LINEAR MEASURE (A NEW TABLE).

Inch, span, cubit, etc., all to be subdivided decimally, or common-fractionally, to suit convenience.

	5 inches	= 1 span	= 5 inches.
	2 spans	= 1 decimal foot	= 10 inches.
	5 spans	= 1 cubit[1]	= 25 inches.
	2½ decimal feet	= 1 cubit	
	4 cubits	= 1 linear standard	= 100 inches.
10 cubits	2½ cubits	= 1 ell[2]	= 1 pole.[3]
	4 ells	= 1 pole	
10 poles	2½ poles	= 1 chain	= 1 acre-side.
	4 chains	= 1 acre-side	
10 acre-sides	2½ acre-sides	= 1 furlong	= 1 metron.
	4 furlongs	= 1 metron	
10 metrons	2½ metrons	= 1 mile[4]	= 1 league.
	4 miles	= 1 league	
	1,000 leagues	= 1 linear axis or polar radius.	

Commencing with the cubit, which is one 10-millionth of the polar radius, the foregoing table is strictly decimal. It is based upon the unit inch, by and through which it connects with all other Anglo-Saxon linear measures founded thereon. It culminates in the polar radius, by and through which it reaches upwards into all solar and sidereal distances. But the system is *doubly* decimal, for running through it we find the following subordinate one: —

SUBORDINATE STANDARD LINEAR MEASURE.

2½ cubits = 62½ inches = one ell.

10 ells	4 ells	= 1 rod	= 1 chain.
	2½ rods	= 1 chain	

[1] 25″ rectified = 25.025 ± English inches; for analogues see International Standard for May, 1883, p. 79.

[2] 62½″; for analogues, see International Standard for May, 1883, p. 88.

[3] 1 pole = 250″; for analogues to which, see tables published in International Standard for May, 1883, and compiled by Jacob M. Clark, C.E.

[4] 1 standard mile = 2,500 cubits; for analogues, see International Standard for May, 1883, p. 89.

10 chains	4 chains	= 1 acre-side	= 1 furlong.	
	$2\frac{1}{2}$ acre-sides	= 1 furlong		
10 furlongs	4 furlongs	= 1 metron	= 1 mile.	
	$2\frac{1}{2}$ metrons	= 1 mile		
	4 miles	= 1 league.		
	1,000 leagues	= 1 semi-axis or polar radius.		

At the first glance, the beauty of the above double decimal arrangement is apparent, and at the second, will be perceived the convenience afforded for general calculation by quarters, halves, fifths, etc.

These common-fractional subdivisions have been found to be so necessary in practice, that even the "metric system" has had to violate its decimal continuity by the admission and interpolation of halves, quarters, etc.

Running through the standard tables, it will be seen that specific *names* are for the most part given to these *common*-fractional parts. Thus:—

1 ell	= $\frac{1}{4}$ rod.
1 chain	= $\frac{1}{4}$ acre-side.
1 furlong	= $\frac{1}{4}$ metron.
1 mile	= $\frac{1}{4}$ league.

The above values make, in fact, a convenient subordinate system by themselves. Thus:—

	4 ells	= 1 rod.
10 rods	= 4 chains	= 1 acre-side.
10 acre-sides	= 4 furlongs	= 1 metron.
10 metrons	= 4 miles	= 1 league.

Furthermore, the halves extend systematically throughout the table, as follows:—

5 cubits	= $\frac{1}{2}$ rod.
5 ells	= $\frac{1}{2}$ chain.
5 rods	= $\frac{1}{2}$ acre-side.
5 chains	= $\frac{1}{2}$ furlong.
5 acre-sides	= $\frac{1}{2}$ metron.
5 furlongs	= $\frac{1}{2}$ mile.
5 metrons	= $\frac{1}{2}$ league.

The above values likewise form harmonious sequences of *practical*, common, and decimal utility. There are two of them which examination will show are alternates. Thus the first one, starting with the half-rod, is as follows:—

5 cubits or 10 half-cubits	=	½ rod.
10 half-rods	=	½ acre-side.
10 half-acre-sides	=	½ metron.
10 half-metrons	=	½ league.

For the alternate we have, —

5 ells or 10 half-ells	=	½ chain.
10 half-chains	=	½ furlong.
10 half-furlongs	=	½ mile.

Now, it is manifest that the decimal subdivision of the unit inch can, as at present, be carried *downward* to the very limits of numerical expression and microscopic appreciation. So, too, beginning at the earth-commensuric inch, we may, for such special purposes as shall be found convenient, run decimally upward into the very largest measures of extension. For instance: —

1″	=	1 unit (to be decimally subdivided).
10 inches	=	1 link.
10 links	=	1 linear standard.
10 linear standards	=	1 measuring line.[1]

Now, we need not tabulate farther than the measuring-line of 1,000.″ Nevertheless, it must be manifest, that, in such a system as we are here developing, the series runs upward harmoniously alongside of, and through the entire system of, rectified linear measure.

Thus, merely for sake of illustration, the above tables may be thrown into rhythm with the whole system, as follows: —

	1 inch	= 1 unit.	
10 inches	2½ inches	= 1 measure	1 link or decimal foot.
	4 measures	= 1 link	
10 links	2½ links	= 1 cubit	1 linear standard.
	4 cubits	= 1 linear standard	
10 linear standards	2½ lin. stand.	= 1 rod	1 measuring line.
	4 rods	= 1 measuring line	

Furthermore, from an inspection it will be seen that two and a half measuring lines equal one acre-side; or, in other words, —

25 linear standards	=	1 acre-side.
250 linear standards	=	1 metron.
2,500 linear standards	=	1 league.
2,500,000 linear standards	=	1 polar radius.

[1] 2½ "measuring-lines" = 2,500″ = 1 side of a square acre; hence 10 "measuring-lines" form the entire perimeter of a square acre.

It is here worthy of note, in passing, that there are the same number (2,500) of linear standards (100″) in a league, as there are of cubits (25″) in a standard mile; and, as we shall see later, of rectified avoirdupois pounds in a standard ton, etc.

Again: based upon the unit inch, and familiar, not only to pyramid students, but to all Anglo-Saxon readers of the Scriptures, we can derive from these rectified measures the ancient sacred measures of the Jews. Thus:—

ECCLESIASTICAL LINEAR MEASURE.

1″	=	1 unit (subdivided decimally).
5″	=	1 span.
5 spans	=	1 cubit.
6 cubits	=	1 reed.[1]

It is the consummate perfection of this system that makes it of such universal application. From the great diversity of its applications, it may seem at first glance somewhat complicated; but the most cursory examination into the *principles* upon which it is based will show that it is just the reverse. We have here presented the system in such a way as to show its full scope, and in so doing may have given the idea of complication; nevertheless, the system itself consists simply of the standard table just given. All of the others, including our well-known linear or long measure, Gunter's measure, superficial measure, cubic measure, etc., are merely subordinate branches of the radical one, based upon the unit, and for special purposes.

Now, is there any need for these subordinate tables? Undoubtedly there is at present. These subordinate systems have sprung up from time to time with the growth of special branches of trade, art, and science. They are not confusing

[1] Mr. Jacob M. Clark, C.E., has shown that the linear measures of the Hebrews (Ezekiel's) were undoubtedly founded upon a radial unit rather than upon a circumferential one. (See Transactions of American Society of Civil Engineers, December, 1882; also International Standard, 1883–84.) It is noticeable in this connection, that Ezekiel's ecclesiastical reed of six cubits (150″) consecutively chords the circle,—i.e., by means of the hexagon,—while it avoids the "impossible problem" of measuring it. Ezekiel thus "vindicated geometry, and proclaimed the divorce of direct measure from circular, and itinerary." This divorce is absolute, from the very nature of things: their forced conjunction is miscegenation, and illegal. The sooner, therefore, we accept the geometry of Ezekiel, and put asunder things which can never have been interiorly conjoined, the sooner we shall accept the facts about us, and square our practice with the things that are.

to those who use them; they are exactly suited to their wants, having been the direct outgrowth of those wants. They have arisen from the necessity of having various standards. The confusion of the past in Anglo-Saxon weights and measures has resulted, not so much from the use of numerous standards, as from the loss of unity in the *units*, by means of which they should have been, and probably were originally, all linked together.

When once more we have rectified our unit inch back to its earth reference, the standard system may be legalized; and with it all the wants of an international, earth-commensuric, and not only thoroughly decimal, but particularly common-fractional system, will at once be realized.

In the mean time the full use of the subordinate systems, rectified at the same time by the adoption of the true unit, can be fully enjoyed by all who are now familiar with them, or may hereafter find them convenient for any special purposes.

It is our NATIONAL duty to perfect *our own* unit, and to found thereon a system that shall be *standard.* But no nation constituted like the branches of the Anglo-Saxon race will ever so forget the grand principles of individual freedom, as to make any system compulsory upon its members. Nor, may we firmly trust, will the Anglo-Saxon race ever so forget its own far-reaching records and traditions, as to exchange a birthright, whereby it compasses the earth with commerce, for a mess of metric pottage.

Such, then, is the Anglo-Saxon system of linear measure,—a system so beautiful and perfect, that, whether it were monumentalized or not at Gizeh, would be worthy of the deepest consideration of the English-speaking race.

This race stands in a peculiar relation to the rest of men. *Its* future is secure: *that of no other people is.* It has not yet adopted the French system. In both Great Britain and America that system has hitherto been only legally *tolerated,* and every effort of its votaries to make it compulsory upon us has thus far most signally failed. Nearly all of the Continental nations of Europe have in the mean while accepted the metric system, and have made it compulsory upon their people. We alone, therefore, among the more enlightened peoples of the day, stand out against it, at present content with our own traditions, and not ready for a change.

But for all Europe to accept the metre does not insure its ultimately universal acceptance at the hands of the rest of the world; far from it, if our vote is against it. The world is not to be European, but Anglo-Saxon.

Leaving the pyramid entirely out of the question, we utterly fail to see *why* — after finding that our own traditional unit of linear measure has been proved to be more ancient than history, of marked personal reference, already in perfect rhythm with the laws of human thought, earth-commensuric, ay, and commensuric with the very universe itself — we should entertain, even for a moment, the thought of adopting one which we know to be just the reverse, and dictated by a people whose customs and manners have so little in common with our own.

We see no reason why a people already outnumbering the inhabitants of all Europe, and who, over the whole face of the globe (where other nations have successively failed to maintain themselves in competition with us), rule as many more millions; a people who actually own the capital, control the industry, and, as a statistical fact, lock up the surplus of the whole earth; whose commerce and practical science has, in a century and a half, changed its face and future, should do aught but take such steps as shall be necessary to PRESERVE and *perfect the system of their own wise ancestors.*[1]

[1] See Appendix A.

III.

CAPACITY MEASURE.

"He layeth up the *depth* in storehouses." — Ps. xxxiii. 7.

"*He weigheth the waters by* MEASURE." — JOB xxviii. 25.

The Coffer in the Great Pyramid. — The Caldron of the Anglo-Saxons. — The Ark, or Laver, of the Hebrews. — The Three Standards Identical. — Our Native Measures over Four Thousand Years Old. — Pyramid Measures according to Professor Smyth. — Noticeable Correspondences. — A Proposed Standard Decimal-Table for International Use. — Further Considerations. — The Table continued Downward — And Upward. — The Resulting System. — Comparison of Results. — Roots of the System. — Comparative Table. — What have we to do with the Metric System in View of Such Great Possibilities? — New Light, and a General Survey of the Rectified Measure.

WERE the command "Measure the *Capacity* of this Coffer" engraved upon its sides, in the symbols of a universal language, the mysterious granite box in the king's chamber of the Great Pyramid could speak no plainer than it now does to the wise of all the earth. It is a most peculiar geometrical solid. It was so exquisitely constructed that its interior capacity — 71,250 pyramid cubic inches — was one-half the volume of its exterior dimensions, and the volume of its thick bottom half that of its sides. Measure it as you will, and employ any unit you desire, still this same relation, so suggestive of capacity, must result. The beauty of this accomplishment will be apparent to all who have ever studied the most renowned mathematical problem of the ancients, — that of "duplicating the cube."

In fact, the coffer *is* a most wonderful measure of capacity. It is a perfect standard: it contains exactly four *original* British "quarters," or one measured ton. It is not a little strange that the "quarter" is now the largest grain-measure upon the statutes of Great Britain. The natural inference is, that, at some time or other, far back into our history, it was a quarter of *something* else, four times greater. Though that something has now dropped out of Anglo-Saxon grain-measure, it must have once held a prominent place therein. The modern British quarter contains 17,745.536 British cubic inches; and four such quarters would contain 71,000 (70,982) British cubic inches,—a quantity still remarkably close to this ancient pyramid-coffer of 71,250 cubic inches. But this cubic volume is also equally close to another and even more significant ancient measure, and to one from which we doubt not we have inherited the caldron itself: we refer to the ark of the covenant, held in such sacred veneration by the people of Israel. Of the absolute contents of the ark, or laver, of the Hebrews, we are not informed; though its capacity may be calculated to a very near degree of accuracy. The Bible gives its outside measures as 2½ cubits long, 1½ cubits broad, and 1½ cubits high. We say outside measures, because the dimensions of the crown of gold which covered it are given in the very same figures; and, had not these been outside measures, the "mercy seat," or cover, would have fallen down inside of the ark. Its outside measures were (in terms of the sacred cubit of 25″), therefore, $62.5'' \times 37.5'' \times 37.5''$. Now we are also left in ignorance as to the thickness of the wood out of which it was constructed. Philo-Israel, in discussing this matter, suggests 1⅞ inches "stuff" as suitable for a well-proportioned box of this capacity. These dimensions would give an inside content of 71,282 cubic inches. Gunner Thomas Henry, R.A., in charge of the officers' workshops at the Royal-Artillery Institution, Woolwich, suggests as rather more suitable working-dimensions a 2-inch bottom, and 1¾-inch sides; which latter material would afford an interior content of 71,213 cubic inches capacity. The mean of these two, undoubtedly very close approximations, is 71,248 cubic inches; or, in all human probability, we have in the ark a box of the same volume as the coffer of the Great Pyramid and the caldron of our forefathers. We may, however, continue the argument a step farther; for we can also examine the molten sea of Solomon,—another and greater measure of capacity, and one also made in terms of the sacred cubit. We shall adhere to the description of it given in 1 Kings vii. 23–26, "that in Chronicles being fragmentary." From this account we find that it was 10 cubits from brim to brim, 30 cubits in circumference, and 5 cubits high. It was round all about, a handbreadth thick, and contained 2,000 baths. We assume nothing in claiming that it was hemispherical in figure; because, 1st, It was round all about; 2d, Its height was half its diameter; 3d, Josephus expressly says it *was* hemispherical. There is also no doubt that the 10 cubits diameter is an outside measure from "brim to brim." Its vertical dimension of 5 cubits being so expressly stated as height, — *not depth,* — must also be taken as outside dimension. Its 30 cubits of circumference, however, must as manifestly be taken as inside dimension; because, 1st, We should otherwise have two conflicting measures of the same thing; 2d, The circle of 30 cubits circumference at the inner lip would leave a margin between the inner and outer edge of the brim of 5½ inches, which, with half an inch allowance for the extra carved work on the brim, may be fairly taken to express the handbreadth thickness of the brazen sea itself. Calculating the volume of such a hemisphere, we find it to be 3,562,070 cubic inches, or, within a minute quantity, 50 times the capacity of the ancient ark of the covenant. The exact size of a fiftieth part of the above-calculated volume is 71,241 cubic inches, or well within the margin required in a casting of such immense size. Moreover, the quantity — 430 cubic inches — by which such a hemisphere actually fell short of the desired amount, 3,562,500 cubic inches, could easily have been compensated for in the ornamental work about the brim.

But the molten sea	=	2,000 baths.
It also	=	50 arks.
Therefore the ark	=	40 baths.
But the laver also	=	40 baths.
Therefore the laver	=	the ark of the covenant.

Thus we have taken a very valuable step towards arriving at the sacred measures of Israel; since we now know that 40 baths = 1 laver, and 50 lavers = 1 molten sea. The exact dimensions of these two measures of Israel serve as checks upon each other; and from them we are not only able easily to fill in the whole biblical series, but to give their capacity in cubic inches, and more fully realize the possible intermediate channel through which these same dimensions may have come down to us from the still more ancient monument of Gizeh. It would thus appear, that the ark was the capacity standard of Israel, and held the same place in the biblical system that the coffer did in that of still earlier days, and that the caldron of our own forefathers did, and does in Anglo-Saxon metrology to-day. As just remarked, the quarter is a grain-measure; and this brings us to one of the best explanations of the meaning itself of the word "pyramid." It is a Greek word, and was first used by Herodotus in describing the wonderful monument which he had visited in his travels. Herodotus probably Hellenized the meaning, if not also the pronunciation, of the word by which the monument was designated. All sorts of significations have been given to it; but there is a very simple one, derived without any contortions whatever, from the Greek word *pyros* ("wheat" or "bread"), and *metron* ("a measure"),—*pyros metron*, or *pyro-met*, or *pyramid* ("the bread-measure"),—the standard International Metric Monument. The pyramid is then a *metric* monument, and the mysterious coffer its standard measure of the *staff* of earthly, and the *symbol* of eternal, life.

Without further discussion, I shall proceed directly to the table of capacity measure, which pyramid students have discovered there. It is a decimal system, superior to any now in use, and grandly Anglo-Saxon. In its scope, it can satisfy the requirements of the minutest scientific analysis, or rise without hiatus to the measure of the universe itself. Professor Smyth arranges the subdivisions of this ancient coffer as follows:—

PYRAMID CAPACITY MEASURE.

Division or number of each denomination contained in the whole coffer.	Intermediate division.	Capacity of each denomination in pyramid cubic inches.	Equivalent weight in pyramid pounds of water.	Name now proposed to be given each successive portion.
1	0	71,250	2,500	coffer.
4	4	17,812.5	625	quarter.
10	2.5	7,125	250	sack.
25	2.5	2,850	100	bushel.
250	10	285	10	gallon.
2,500	10	28.5	1	pint.
25,000	10	2.85	0.1	wine-glass, or fluid oz.
250,000	10	0.285	0.01	teaspoon, or fluid dr.
2,500,000	10	0.0285	0.001	ten drops.
25,000,000	10	0.00285	0.0001	drop.

He then subjoins the following table, to illustrate how closely our present Anglo-Saxon measures of capacity still correspond with their ancient source and standard.

PYRAMID AND BRITISH CAPACITY MEASURES COMPARED THROUGH THE TEMPORARY MEDIUM OF ENGLISH CUBIC INCHES, APPROXIMATELY.

Pyramid.		English.	
Coffer	71,463.750	4 quarters	70,982.144
Quarter	17,865.938	1 quarter	17,745.536
Sack	7,146.375	1 sack	6,654.576
Bushel	2,858.550	1 bushel	2,218.192
Gallon	285.855	1 gallon	277.274
Pint	28.585	1 pint	34.659
Ounce, in wine-glass	2.858	1 ounce, apothecaries'	1.733
Dram, or teaspoon	.286	1 dram	.217
Drop	.003	1 drop	.004

Professor Smyth then gives a series of tables to show that these measures correspond now with the pyramid in quarter, sack, and bushel, for many parts of the world.

But there are many other correspondences between these ancient measures and our own well-known Anglo-Saxon ones, which do not appear upon the surface of this table. For instance, in the United States the pint of liquid measure equals 28.875 cubic inches, and is based upon the *old* British wine-gallon. The present British pint weighs a *pound and a quarter.* An ancient Anglo-Saxon pound, however, was an exact pint, and contained $27.73 \pm$ cubic inches, one-sixteenth of which, or 1.736 cubic inches, apparently accounts for the apothecaries' ounce as above (1.733) given. Now, by inspection it appears probable that the apothecary ounce of 1.733 cubic inches was originally derived by taking the one 10-thousandth of the quarter, — a supposition which becomes irresistible when we go to pyramid originals. Thus the pyramid "quarter" of 17,865.938 British cubic inches $\div$ 10,000 $= 1.78659 \pm$ British cubic inches; and if we multiply by 16 (the subdivision for avoirdupois pounds) we obtain 28.585 British cubic inches; i.e., the pyramid avoirdupois pound; while the same quantity multiplied by 12 gives us 21.438 British cubic inches, or what was undoubtedly the original troy and apothecaries' pound (now only $1.733 \times 12 = 20.796$ British cubic inches).

From the foregoing considerations, we may rest assured, that, while the coffer is the grand standard of both pyramid and Anglo-Saxon capacity measures, the measured *ounce*, or the one 10-thousandth of its quarter, and the gill, or one 10-thousandth of the coffer itself, are the units by means of which its several kinds of *pints* and measured *pounds* have been derived and are connected, and are still to be preserved and protected.

This is the same view we have already taken of the inch in linear measure; namely, that it is the *unit* whence, for the purpose of various trades, the foot, the yard, and the ell were all obtained.[1]

I shall now invite attention to the following *re*-arrangement of the pyramid-capacity table, into which I have introduced a number of other well-known English measures, and by means of which the double decimal nature of the system becomes instantly apparent. It will be noticed, however, that this table differs from that of Professor Smyth, in that it brings into the system several measures which he overlooked, while it leaves intact the values of most all of those which he did employ. In one or two cases, however, for obvious reasons, I have also re-arranged some of his proposed designations.

STANDARD TABLE OF PYRAMID CAPACITY MEASURES, FROM THE OUNCE UP TO THE COFFER.

10 meas. ounces . .	4 meas. oz.	= 1 gill	= 1 stand. lb. meas.
	2½ meas. gills	= 1 standard lb. meas.	
10 standard lb. meas.	4 stand. lbs.	= 1 quart.	= 1 pottle.[2]
	2½ quarts	= 1 pottle.	
10 pottles	4 pottles	= 1 sack.	= 1 bushel.
	2½ sacks	= 1 bushel.	
10 bushels	4 bushels	= 1 coomb.[2]	= 1 quarter.
	2½ coombs	= 1 quarter.	
4 quarters			= 1 coffer.

[1] The word "pound" is derived through *pondus* ("a weight"), from the Latin verb *pendere* ("to weigh"), and may be of any convenient number of ounces, depending upon the various requirements of art and trade. It simply means *a weight* (and *pint* an equivalent *measure*), and by every class of workmen was originally applied as a designation to that particular measure or weight (i.e., number of units) which *they* handled most frequently. Thus, in United-States liquid measure we have the *pint* of 28.875 cubic inches, and in dry measure of 33.600 cubic inches, and, finally, in British imperial measure (both liquid and dry), one of 34.659 cubic inches. It almost seems impossible that such diverse measures can have had a common unit, nevertheless I doubt not we shall be able to establish it as an incontrovertible fact before we close the topic.

[2] Saxon terms at present in use. See British imperial measure.

An examination of the foregoing table will show that it is strictly decimal, while it is at the same time conveniently arranged for the purposes of every-day life, which demand quarters, halves, eighths, etc., even more emphatically than they do the employment of tens and hundreds.

It will be noticed, however, that there is a subordinate system of tens running through this table, by means of which even the subdivisions themselves are also decimally correlated. Thus from the gill, of four ounces, upward, we have the following arrangement: —

4 ounces	= 1 gill.
10 gills	= 1 quart.
10 quarts	= 1 sack.
10 sacks	= 1 coomb.
10 coombs	= 1 coffer.

I shall now ask your attention to the cubical capacity (in inches) of the various measures above given, with a view to determining how nearly they are related to our present Anglo-Saxon measures.

PYRAMID CAPACITY MEASURES COMPARED THROUGH THE TEMPORARY MEDIUM OF ENGLISH CUBIC INCHES.

(PART I. — OUNCE TO QUARTER.)			
Measured Ounces.	Subdivisions.		British Cubic Inches.
1	1 measured ounce	= 1 unit of capacity.	1.7865938
10	10 measured ounces	= 1 measured pound.	17.865938
100	10 measured pounds	= 1 pottle.	178.65938
1,000	10 pottles	= 1 bushel.	1,786.5938
10,000	10 bushels	= 1 quarter.	17,865.938
(PART II. — GILL TO COFFER)			
Gills.	Subdivisions.		British Cubic Inches.
1	1 gill	= 4 units of capacity.	7.1463750
10	10 gills	= 1 quart.	71.463750
100	10 quarts	= 1 sack.	714.63750
1,000	10 sacks	= 1 coomb.	7,146.3750
10,000	10 coombs	= 1 coffer.	71,463.750

The above table presents the radical or "standard" pyramid system under the temporary medium of our *present* cubic inches. With its primary and secondary units, — the measured ounce and gill, — we are now particularly concerned; since from them it can be clearly established that avoirdupois, apothecary, troy, liquid, dry, and even the British imperial measures, must have been originally derived. Thus sixteen of these pyramid unit ounces equal 28.5855 English cubic inches, and form the basis for the capacity of the common or avoirdupois pound. Trautwine says, "The standard of the avoirdupois pound, which is the one in common use, is the weight of 27.7015 cubic inches of pure distilled water, at its maximum density, or about 39.2° Fahrenheit, in latitude London, sea-level barometer 30″." This same amount of water at the mean temperature of the earth (i.e., pyramid standard) would gain about .016 ± in bulk (i.e., .443 ± British cubic inches), and become at least 28.14 ± cubic inches.

Turning now to United-States liquid measure, we find that its pint (the old "liquid pound") contains 28.875 cubic inches; which is proportionately about as much in excess of sixteen of our pyramid units as the avoirdupois pound is short thereof. The pint of the British imperial (liquid and dry) measure is one and a fourth pounds (avoirdupois) of water, or almost equal to twenty of our unit ounces, allowing for mean temperature, and loss of traditions. Now it is manifest that these, the true bases of avoirdupois, liquid, and imperial measures, once established, their whole tables follow as a matter of course, and connect with the radical system at Gizeh, through the real unit 1.7865+ English cubic inches.

In troy and apothecaries' weights the pounds are the same, and, by measure, are each closely equal to twelve of our unit ounces, or to $(12 \times 1.78659 \pm) = 21.4390 \pm$ English cubic inches, mean temperature, pressure, etc.

From an examination of the gill as a cubic measure of capacity, similar relations will be found to exist. Thus Trautwine puts the modern gill at 7.21875 cubic inches, which is very close upon that of our table (7.1463). Allowing for increase due to temperature, little, if any, difference would exist. Four of these gills give him the pint of liquid measure, 28.875 cubic inches, and connect directly with the capacity of the avoirdupois pound, etc. The minute discrepancies which may be detected in these comparisons are nothing more than should have been expected from our loss of traditions, and the remoteness of the time when they were last compared with their grand standards. Moreover, a large proportion of these discrepancies also vanish upon equalizing the temperature. The most astonishing consideration, however, is that they should ever have agreed so closely.

In the table of pyramid measures of capacity which we have just considered, we chose the limits, — from the measured ounce and the gill, to the quarter and coffer — respectively, as those comprising the requirements of the more ordinary phases of human life. The pyramid system, however, extends harmoniously both above and below these limits, so as not only to satisfy the minuter requirements of scientific analysis, but equally to comprehend those of the gigantic commerce of the dawning international future.

Thus, the decimal subdivisions of the ounce and gill may be continued almost indefinitely downward, as follows: —

TABLE OF PYRAMID CAPACITY MEASURES, EXTENDED DOWNWARD.

10 milliminims .	4 milliminims 2½ atoms	= 1 atom. = 1 centiminim.	= 1 centiminim.
10 centiminims .	4 centiminims 2½ drops	= 1 drop. = 1 deciminim.	= 1 deciminim.
10 deciminims . .	4 deciminims 2½ doses	= 1 dose. = 1 minim.	= 1 minim.
10 minims . . .	4 minims 2½ tablespoons	= 1 tablespoon.[1] = 1 measured ounce.	= 1 measured ounce.
4 measured ounces			= 1 gill.

The subordinate system of the above table is as follows: —

10 atoms	= 1 drop.
10 drops	= 1 dose.
10 doses	= 1 tablespoon.
10 tablespoons	= 1 gill.

[1] Usually 8 to 10 teaspoons, or 4 to 6 dessert-spoons, or 2 to 3 tablespoons of large capacity, may be reckoned at one ounce; or at ratio of 9 : 5 : 2½ per ounce.

On extending the system upward from the quarter and coffer, some such series as the following must necessarily result: —

TABLE OF PYRAMID CAPACITY MEASURES, EXTENDED UPWARD.

10 coffers	$2\frac{1}{2}$ coffers = 1 bin. 4 bins = 1 car-load.	= 1 "standard," or car-load.
10 car-loads . . .	$2\frac{1}{2}$ car-loads = 1 lighter. 4 lighters = 1 barge.	= 1 barge.
10 barges	$2\frac{1}{2}$ barges = 1 ship-load. 4 ship-loads = 1 elevator.	= 1 elevator.
10 elevators . . .	$2\frac{1}{2}$ elevators = 1 wharf. 4 wharves = 1 district.	= 1 district.
10 districts	$2\frac{1}{2}$ districts = 1 section. 4 sections = 1 ark.	= 1 ark, or harbor.

The above likewise involves the following subordinate one: —

10 bins . .	=	1 lighter.
10 lighters	=	1 ship-load.
10 ship-loads	=	1 wharf.
10 wharves	=	1 section.
4 sections	=	1 ark, or harbor.

The pyramid system is, in fact, as unique from the stand-point of capacity measure as we have already found it to be from that of linear measure. Though continuous throughout, it may be naturally subdivided into six separate tables, severally comprised between the following limits: —

TABLE I. — Milliminim to measured ounce. TABLE II. — Atom to gill.	For scientific use in arts, professions, and trades.
TABLE III. — Measured ounce to quarter. TABLE IV. — Gill to coffer.	For ordinary use of every-day life.
TABLE V. — Quarter to section. TABLE VI. — Coffer to ark.	For commercial and international use.

Each table is decimally subdivided, and is intimately related to its alternates by the simplest common fractions ($\frac{1}{4}$, 4, $\frac{1}{2}$, 2, $\frac{2}{5}$, 25, etc.); and reductions from one measure to another may be accomplished in the fewest possible figures, and without endless fractions.

Tables I. and II. are concerned particularly with processes of the more exact sciences and minute analysis. Tables III. and IV. comprise the measures of familiar, every-day life; while the last two, V. and VI., are capable of satisfying the demands of the most gigantic commercial enterprise that shall arise in the day of universal peace and national intercourse.

Let us now view the central portion of this system — that comprised between the measured ounce and coffer — in the light of its own integrity, as expressed in pyramid cubic inches.

PYRAMID CAPACITY MEASURES COMPARED THROUGH THE MEDIUM OF PYRAMID CUBIC INCHES.

(PART I. — OUNCE TO QUARTER.)			
Measured Ounces.	Subdivisions.		Pyramid Cubic Inches.
1	1 measured ounce.		1.78125 (exact).
10	10 measured ounces	= 1 measured pound.	17.8125 "
100	10 measured pounds	= 1 pottle.	178.125 "
1,000	10 pottles	= 1 bushel.	1,781.25 "
10,000	10 bushels	= 1 quarter.	17,812.5 "
(PART II. — GILL TO COFFER.)			
Gill.	Subdivisions.		Pyramid Cubic Inches.
1	1 gill.		7.1250 (exact).
10	10 gills	= 1 quart.[1]	71.250 "
100	10 quarts	= 1 sack.	712.50 "
1,000	10 sacks	= 1 coomb.	7,125.0 "
10,000	10 coombs	= 1 coffer.	71,250 "

The foregoing is not only the radical table of the entire system, but from it may be renewed the various special ones, such as avoirdupois, liquid, etc., now in use. For instance, sixteen of the above unit measured ounces give us exactly 28.5 pyramid cubic inches as the capacity (in pure water at mean earth temperature, pressure, and hygroscopic state, 50° pyramid) of an avoirdupois pound, from which the whole avoirdupois sequence may be duly determined. Now, the *rationale* of taking sixteen of these unit ounces to form the capacity of the avoirdupois pound becomes at once apparent upon reviewing the

[1] Four such quarts = 285. cubic inches, the present milk-gallon of New Hampshire.

United-States liquid measure, where four gills are taken for a pint. Taking, therefore, four pyramid gills of 7.125 pyramid cubic inches each, we have one pyramid pint = 28.5 pyramid cubic inches, the same capacity as the pyramid avoirdupois pound, in pure water. Hence, as four ounces make a gill, and four gills a pint, sixteen measured ounces make both a pint and a measured avoirdupois pound. And avoirdupois becomes at once related to liquid measure, and the two, through the radical ounce (1.78125), to the whole pyramid system. In the British imperial system, since its pint is taken to be $1\frac{1}{4}$ avoirdupois pounds, a pyramid gill of 7.125 inches, added to the pyramid avoirdupois pound (28.5), gives its capacity. Thus, $28.5 + 7.125 = 35.625$ pyramid cubic inches = one rectified British imperial pint; in terms of which, if the imperial system be *renewed* and *perfected*, it will become at once, not only earth-commensuric, but directly convertible, through the medium of the unit pyramid measured ounce (1.78125), into the terms of any other rectified Anglo-Saxon system. Again: since the basis (Trautwine) of United-States dry measure "is the old British Winchester struck bushel" (given as 2150.42 English cubic inches), by connecting it with the pyramid system we shall render all of its elements duly convertible into any other Anglo-Saxon measures. Now, inspection of the pyramid system shows that the original basis of this old "struck bushel" must have consisted of 350 gills, with the "heap," usually one-seventh, scraped off; i.e., 350 gills *struck* to 3 sacks, 30 quarts, or 300 gills; i.e., of 2,137.5 pyramid cubic inches. The use of this as the basis of subdivision will consequently *perfect* the system, and render it convertible in simple terms and whole numbers.

Finally: the pounds of troy and apothecary measures consist of twelve ounces each; i.e., pyramid standard ounces. Their true capacity of pure water is, therefore, $1.78125 \times 12 = 21.375$ pyramid cubic inches; or, in other words, it is exactly one-hundredth of a "struck bushel."

Thus, through the medium of the radical pyramid, "measured ounce," and "gill," every one of the Anglo-Saxon tables of capacity and weight (i.e., measure and capacity of the water-equivalent) becomes directly related to all the others.

To recapitulate, the roots of the various systems as rectified are as follows: —

				Pyr. cubic inches.
1 "unit" pyramid measured ounce			=	1.78125
1 standard pyramid gill			=	7.125
1 pyramid avoirdupois pound	=	16 pyramid ounces	=	28.5
1 pyramid pint (liquid, U.S.)	=	4 pyramid gills	=	28.5
1 pyramid struck bushel (dry, U.S.)	=	300 pyramid gills	=	2,137.5
1 British imperial pint (dry and liq.)	=	20 pyramid ounces	=	35.625
1 troy or apothecary pound	=	12 pyramid oz. or pyramid gills	=	21.375

These values are all based upon the contents of the coffer in pyramid cubic inches, being 71,250. Proportional values, of course, result when we take the contents of this standard in British cubic inches.

In order now to set forth what will be the nature of the connection between the several systems of special Anglo-Saxon measures and the standard system, — that of the Great Pyramid, — when the former shall have the units duly *perfected* thereby, we subjoin the following table. By reading it across, there will be seen at a glance the equivalent of any designated measure in terms of the rectified unit of each particular system. Thus the coffer itself, of 71,250 pyramid cubic inches capacity, is seen to be equal to 4,000 standard (or 10 oz.) pounds, 2,500 avoirdupois (or 16 oz.) pounds, or liquid-measure pints, 3,333⅓ troy or apothecary (12 oz.) pounds, 2,133⅓ United-States dry measure (or 18.75) pounds, and 2,000 British imperial (20 oz.) pints, in which, throughout, the *ounce* considered is the standard "unit" (or one 10-thousandth of the quarter taken for capacity at 17,812.5 pyramid cubic inches).

PYRAMID CAPACITY MEASURE COMPARED WITH RECTIFIED ANGLO-SAXON MEASURE.

Names of successive portions.	Intermediate subdivisions.	Capacity in pyramid cubic inches.	Equivalent capacity in standard pyramid lbs. (of water, 17.8125 pyramid cubic inches at mean temperature, pressure, etc., per lb.) of 10 "unit" ounces.	Equivalent capacity in avoirdupois pounds, or U. S. liquid pints of 28.5 pyramid cubic inches per pint, or pounds of 16 "unit" ounces.	Equivalent capacity in troy and apothecaries' pounds of 21.375 pyramid cubic inches per pound of 12 "unit" ounces.	Equivalent capacity in U. S. dry-measure pints of 33.3984375 pyramid cubic inches per pint of 18.75 "unit" ounces.	Equivalent capacity in British imperial measured pints of 35.625 pyramid cubic inches per pint of 20 "unit" ounces.
Ark	1	7,125,000,000	400,000,000	250,000,000	333,333,333.33	213,333,333.33	200,000,000
Section	4 } 10	1,781,250,000	100,000,000	62,500,000	83,333,333.33	53,333,333.33	50,000,000
District	10 { 2.5	712,500,000	40,000,000	25,000,000	33,333,333.33	21,333,333.33	20,000,000
Wharf	4 } 10	178,125,000	10,000,000	6,250,000	8,333,333.33	5,333,333.33	5,000,000
Elevator	10 { 2.5	71,250,000	4,000,000	2,500,000	3,333,333.33	2,133,333.33	2,000,000
Ship-load	4 } 10	17,812,500	1,000,000	625,000	833,333.33	533,333.33	500,000
Barge	10 { 2.5	7,125,000	400,000	250,000	333,333.33	213,333.33	200,000
Lighter	4 } 10	1,781,250	100,000	62,500	83,333.33	53,333.33	50,000
Car-load	10 { 2.5	712,500	40,000	25,000	33,333.33	21,333.33	20,000
Bin	4 } 10	178,125	10,000	6,250	8,333.33	5,333.33	5,000
Coffer	10 { 2.5	71,250	4,000	2,500	3,333.333	2,133.333	2,000
Quarter	4 } 10	17,812.5	1,000	625	833.333	533.333	500
Coomb	10 { 2.5	7.125	400	250	333.333	213,333	200
Bushel	4 } 10	1,781.25	100	62.5	83.333	53.333	50
Sack	10 { 2.5	712.5	40	25	33.333	21.333	20
Pottle	4 } 10	178.125	10	6.25	8.333	5.333	5
Quart	10 { 2.5	71.25	4	2.5	3.333	2.133	2
Pound	4 } 10	17.8125	1	.625	.833	.533	.5
Gill	10 { 2.5	7.125	.4	.25	.333	.213	.2
Ounce	4 } 10	1.78125	.1	.0625	.0833	.053	.05
Tablespoon	10 { 2.5	.7125	.04	.025	.033	.0213	.02
Minim	4 } 10	.178125	.01	.00625	.0083	.0053	.005
Dose	10 { 2.5	.07125	.004	.0025	.0033	.00213	.002
Deciminim	4 } 10	.0178125	.001	.000625	.00083	.00053	.0005
Drop	10 { 2.5	.007125	.0004	.00025	.00033	.000213	.0002
Centiminim	4 } 10	.00178125	.0001	.0000625	.000083	.000053	.00005
Atom	10 { 2.5	.0007125	.00004	.000025	.000033	.0000213	.00002
Milliminim	4	.000178125	.00001	.00000625	.0000083	.0000053	.000005

Not the least remarkable feature in the foregoing table is the *wholeness* in which the numerical exchange of values comes out. The passage from one system to another is thus a process of the very simplest character.

From the foregoing consideration of the capacity measures now in use throughout the Anglo-Saxon world, it is evident that what the English-speaking people need, in order to obtain a correct and earth-commensuric system, is not by any means an abolishment of its present measures. *All that is necessary is that the units of the various systems shall be severally based back once more upon what is now clearly shown to have been their original common* UNIT.

This once accomplished, the various trades may continue in the unmolested employment of those particular multiples of measure and capacity which have been the result of such long (at least four thousand years) experience. No terms will be lost, no confusion inaugurated, by a wholesale change, as it were, of "times and seasons;" and the *small corrections* introduced will be actually *well within the margin now allowed in daily trade and commerce;* while for purposes of mutual comparison and intercourse, the most desirable ends will thus be realized.

What, then, indeed, have Anglo-Saxons to do with the metric system in the face of such possibilities as are now before them, affecting the absolute perfection of their own? Why, having preserved our "traditions" so long, or, rather, having had them preserved *for* us so wonderfully, should we now discard them, just as their beauties are being fully discovered? What has the metric system to offer in exchange for what we now have, and for what we *may* have by perfecting our own? Its decimal system is absolutely its only claim; and this is entirely cancelled against the doubly decimal arrangement of what the Anglo-Saxon inherits in the Great Pyramid, *and in the earth on which he lives.* Eliminate its decimal arrangement from the metric system, and nothing whatever remains but an erroneous basis, a miscalculated standard, a confusion of unknown terms, and a set of unscientific atmospheric conditions, all of which are absolutely hostile to the every-day and practical requirements of man.

IV.

WEIGHT MEASURE.

Let Him who hath weighed the mountains in scales, weigh me in balances of justice.— JOB xxxi. 6; ISA. xl. 12.

Relation of Capacity to Weight.—The Standard Chamber at Gizeh.—Present Anglo-Saxon Weight-Measures.—Their Lack of Harmony.—Pyramid Weight-Measure.—Its Ultimate and Grain used to unify the Saxon Measures.—The Use of a Greatest Common Divisor.—The Origin of British Metrology.—The Saxon Tables as Unified and Rectified.—A Proposed Decimal Standard Table of Weights for International Purposes.—The Constants of the System.—The Peculiar Subdivision of the Ounce.—Wonderful Possibilities of Complete Resolution.—The *Last Drop* in the Coffer, the Last Atom of the Caldron.—The Cubic Inch in Grains and Ultimates.—Mean Density of the Earth.—Its Use in connecting Weight and Capacity.—Its Value in Grains and Ultimates a Most Important Number.—Can the Metric System accomplish Such Feats?—Manufacture of Weights out of Standard Material, Lead, Iron, etc.—The Ancient Metrology of Israel Part of this Earth-Commensuric and Rectified Saxon System.—The Wonderful *Cubical* Properties involved.—The Challenge to the Metric System.—The French System examined.—Its Only Feature of Value, the Decimal, is not its Own.—The Duty of the Few who use the Metric System.—The Duty of the Many who use the Saxon.

INTIMATELY connected with the subject of capacity measure is that of weight. They are indeed so necessarily correlative, that we have already found it impossible to compare the several units and systems of capacity measure, without partially anticipating our present topic.

Now, the most natural method of obtaining a unit of weight is to fill one of capacity with some convenient substance, and assume the weight of the result as the quantity sought. It is universally customary to employ water at some standard temperature and pressure for this purpose. In fact, water is employed as the standard of comparison for all weight whatsoever, since it is taken as the unit of specific gravity itself.

If the coffer, therefore, which we have already found to be so remarkable a capacity measure, be now filled with pure water, it will be instantly converted into a standard of weight; and in this, its new employment, we shall find it capable of affording us even more astonishment than ever.

The temperature and barometric pressure in the king's chamber we already know to be those most convenient for all ordinary human avocations. They are the mean of all the earth, more easily maintained, and less frequently varied from, than any other standard points that could possibly have been agreed upon. Now, 71,250 pyramid cubic inches of water, under the circumstances presented in the king's chamber, weigh exactly 2,500 *rectified* avoirdupois pounds, or liquid-measure pints. This fact recalls most vividly the ancient Anglo-Saxon rhyme, —

"A pint's a pound,
The world around."

If Anglo-Saxon weights and measures, duly *corrected* by the pyramid system, are to survive in the "battle of the standards," now being so fiercely waged, then the above distich is deeply prophetic. Let us proceed, therefore, to examine the three systems now in common use, — troy, apothecary, and avoirdupois *weights*, — in the light afforded by the pyramid, and see if they are really worthy of our support in the struggle now in progress.

In Trautwine's "Engineer's Pocket-book" we learn, that, for the United States and Great Britain, these weights are as follows: —

Troy.	— 1 pound =	12 ounces =	240 pennyweights		= 5,760 grains.
Apothecary.	— 1 pound =	12 ounces =	96 drams =	288 scruples	= 5,760 grains.
Avoirdupois.	— 1 pound =	16 ounces =	256 drams		= 7,000 grains.

We also read in this manual, that at present "the troy ounce is greater than the avoirdupois, but the troy pound is less than the avoirdupois; while in troy and apothecary weights, the grain, ounce, and pound are the same." "An avoirdupois pound = 1.21528 troy pounds, and an avoirdupois ounce = .911458 of a troy ounce. Thus it is manifest that the grain is the only measure *common* to all of them; and this as a divisor is now of little value, save for the temporary purpose of mutual reduction.

Its present value was arbitrarily chosen by Parliament at the close of the eighteenth century, in a moment of too hasty legislation, when 24 *artificial* grains (i.e., "so called, but of no known variety of plant employed for breadstuff") were put for what had formerly been the number, 32, that went to make up a pennyweight, troy. We say the present grain is of little value save as a common divisor for the temporary purpose of reduction. This is because it is based upon the erroneous idea, that the ounces or units in the three systems, as well as the pounds, are not the same. Now, we have already shown, while discussing the *capacities* of these various systems, that the standard unit of capacity, 1.78125 pyramid cubic inches, at mean temperature and pressure, was undoubtedly the original basis of these measures, and that it is the one by which their rectification should now be effected. This once accomplished, it is manifest that the value of the present artificial grain as a common divisor will instantly disappear. But its retention, even at present, involves the perpetuation of an inconvenience. For example, while the table of troy weight begins "24 grains make a pennyweight," and that of apothecary weight commences "20 grains make a scruple" (i.e., both even and convenient numbers), the table of avoirdupois weight (the very one of all others that should be even, as it is the most universally employed) opens with the awkward statement that 27.34375 grains make a dram.

The systems are *not harmonious;* and it is on account of such evident flaws as the above, that the now mutilated Anglo-Saxon weights and measures have so many opponents. Let, however, these systems once be *rectified back to their ancient* standard, and no system that can be conceived of will be able to maintain itself against them.

In the following table, commencing with the ounce-weight and ending with the ton, the same system has, of course, been followed as in the capacity measures. Designations have simply been altered to suit the subject (weight) under consideration. It may also be noticed here, that, as in the case of the capacity measures, this same system can be extended both upwards and downwards to correspond therewith. We shall, however, at present content ourselves with the consideration of this table alone, and of the various Anglo-Saxon systems thence derived through the medium of its unit ounce.

STANDARD TABLE OF PYRAMID WEIGHT-MEASURE.

(FROM OUNCE TO TON.)

10 ounces (weight)	4 oz. (weight)	= 1 gill weight.	= 1 stand. pound wt.
	2½ gill weights	= 1 stand. pound wt.	
10 pounds (weight)	4 lbs. (weight)	= 1 quart weight.	= 1 stone.
	2½ quart weights	= 1 stone.	
10 stone	4 stone.	= 1 sack weight.	= 1 hundred weight.
	2½ sack weights	= 1 cwt., or quintal.	
10 hundred weight	4 cwt.	= 1 wey.	= 1 thousand wt.
	2½ weys	= 1 thousand weight.	
4 thousand weight			= 1 ton = 1 coffer.

The subordinate decimal system is as follows:—

4 ounces	= 1 gill weight.
10 gill weights	= 1 quart "
10 quart weights	= 1 sack "
10 sack weights	= 1 wey.
10 weys	= 1 ton.

In the above tables the ounce is the weight of a unit of capacity,— 1.78125 pyramid cubic inches filled with pure water at mean earth temperature (50° pyramid), pressure (50° pyramid), and hygroscopic state (50° pyramid); and the gill-weight that of four such units, or 7.125 pyramid cubic inches.

We have already tabulated under the head of capacity measures the comparative values, in terms of the weight of their own rectified units, of the troy, avoirdupois, and apothecary subdivisions, which correspond to the foregoing table.

It is now necessary to determine the proper number of grains originally contained in the "pound-weights" of these systems as now rectified; since, as before mentioned, by the very process of rectification, the *numbers* of arbitrary grains contained in these pounds will lose all their present significance.

From various considerations, ramifying out into too many subordinate subjects to be considered here, it seems probable that the number of *real* ultimates originally contained in the ancient coffer of 71,250 pyramid cubic inches capacity, was 25,000,000 $= (5{,}000)^2 = (25 \times 2 \times 100)^2 = (50 \times 100)^2$. Let us therefore apply this number, and see if it will harmonize the several systems, and assist us in determining their greatest common divisor. Let x = 25,000,000 ultimates. Then there having been found to be 4,000 standard (i.e., 10-oz.) pounds in the coffer, we have —

1 pyramid standard pound weight $= \frac{x}{4,000} =$ 6,250 "ultimates."

And for similar reasons

1 pyramid troy or apothecary pound weight $= \frac{x}{3,333\frac{1}{3}} =$ 7,500 "ultimates."

1 pyramid avoirdupois pound weight $= \frac{x}{2,500} =$ 10,000 "

Furthermore, dividing 71,250 pyramid cubic inches by 25,000,000, we learn that one such standard ultimate contains exactly .00285 pyramid cubic inches of pure water; i.e., the unity material at mean temperature, pressure, hygroscopic state, etc., or is $\frac{1}{2500}$ of a gill of water. The above numbers of standard ultimates per pound give us for each of the several systems the following numbers: —

In troy weight	31.25	= $31\frac{1}{4}$	(exact) ultimates	= 1 pennyweight.
In apothecary weight	26.04166	= $26\frac{1}{24}$	" "	= 1 scruple.
In avoirdupois weight	39.0625	= $39\frac{1}{16}$	" "	= 1 dram.

At the first inspection, the above figures appear to be somewhat objectionable, as every one of them involves a fraction; while of those now in use only one (the avoirdupois) does so. Such was at first my own impression after calculating them. Though all of them are *exact*, they involve decimal terminations which are the very things we wish to avoid in a (to be) popular table of measures. Closer study, however, revealed to me their hidden beauty. The greatest common divisor of these three quantities is $1.30208\frac{1}{3}$ ultimates; the cubic capacity of which is .0037109375 (exact) pyramid cubic inches of water, or, in other words, exactly $\frac{1}{1920}$ of an avoirdupois gill of water.

I shall now retain the present name of this common divisor, and designate the quotients thus obtained as *grains* (i.e., $\frac{1}{1920}$ *part* of an avoirdupois gill). Dividing each of the above numbers by it (i.e., by $1.30208\frac{1}{3}$ ultimates), we have, —

For troy weight	24 grains	= 1 pennyweight (as before).
For apothecary weight	20 "	= 1 scruple (as before).
For avoirdupois weight	30 "	= 1 dram (as now *rectifiea*).

Hence we have rediscovered the *rationale* of the traditional subdivision of the pennyweight into twenty-fourths, and of the scruple into twentieths, and, in rectifying the systems by comparison with their pyramid standards, have been enabled to relegate into oblivion the awkward number of 27.34375 artificial grains with which the avoirdupois table now opens, and substitute therefor the simple number 30.

These numbers (24, 20, and 30) result from the very nature of things, and are manifestly of great importance in conveniently continuing the final subdivisions down through halves, quarters, thirds, etc., to the very terminals demanded by the arts, trades, and professions specially concerned with the use of each several subdivision of the general pyramid, earth-commensuric, or Anglo-Saxon systems. It is thus manifest, that what we now *call* a grain in our present apothecary and avoirdupois weights is not the real and original *ultimate* of weight-measure; but it is the result of dividing the terminal number of the several tables expressed in original ultimates by their greatest common divisor, and that its introduction into the systems was, and is, the means of clearing them of fractions.

Troy, avoirdupois, and apothecary weights pass back together into the most remote Anglo-Saxon times. From *à priori* reasoning, it is clear, that, in their origin (as we have now actually demonstrated), they must have had a common unit. It would be a most difficult undertaking to trace their actual history, to tell when they severally broke off from the parent stock, why they did so, and which of them is the oldest system. We could probably trace the ounce back to the most remote times. Like the inch, its origin is shrouded in the deepest mystery, — a mystery all the more tantalizing because it turns out to be so consummately perfect a unit. Let us put the origin of the troy system, for the sake of argument, back to the time of the Norman Conquest. Now, which is the more probable, — that the jewellers and goldbeaters actually originated a whole system, unit, standard, and all, or that they merely took the unit of their ancestors, and arranged its subdivisions and multiples to suit the demands of their own trade; i.e., that they adopted a new standard, and retained the old unit? Manifestly the latter. Otherwise there would have been no means of comparison: otherwise an ounce of gold would not have been intelligible to their purchasers, who lived upon ounces of bread. It was absolutely necessary, that somewhere in each of the elementary systems, as they successively originated, there should have been a connecting link with what had gone before. It is one thing to adopt a standard; as, for instance, to say to-day that hereafter twenty-five inches shall be called a cubit (sacred or ecclesiastical), and that all church-architecture shall be based upon it: it is quite another to make an entirely new unit also; as, for instance, the metric system. In the one case, we preserve our traditions, and work intelligently among our neighbors: in the other, we ruin all things with a deluge. To adopt the second's pendulum, or some other constant thing, as a standard capable of perpetual renewal, and to ordain that it shall be interpreted in inches, a well-known previous unit, with a view of squaring and renewing feet, yards, and ells forever at a constant value, is an act of wisdom; but to break away from every tradition, and make all things new by *curving* them against an arbitrary—ay, even an imaginary, impossible, and absolutely indeterminate—thing, is an act of folly, worthy only of the dreadful "age of revolution." The ratio exists, and can be found, by which the second's pendulum, at any latitude, is a constant function of the polar axis; and as, *cæteris paribus*, the pendulum for a constant latitude must be a constant length, we can by its means say what is the cubit or the one 10-millionth of the polar radius. This alone is a standard fit for all times and places, and is eternal in its permanence; and a twenty-fifth of such a standard is the Anglo-Saxon inch and unit.

But our discovery throws a flood of light in another direction. Since we have now established that the true numbers of the real ultimates actually contained in the various Anglo-Saxon and pyramid pounds are much greater than those now in use, the question instantly arises, How are we to account for the numbers 4,800, 5,760, and 7,680, which have come down to us traditionally as part and parcel of the Anglo-Saxon systems, and have so long been used as grains? The answer is a very simple one. These numbers express the *values* of the several Anglo-Saxon pounds (standard, troy and apothecary, and avoirdupois), in terms of their greatest common divisor

(.00285 × 1.30208$\frac{1}{3}$ = .0037109375 exact cubic inches of water at standard circumstances, = $\frac{1}{1920}$ of an avoirdupois gill = $\frac{1}{480}$ of a unit ounce).

Will any one maintain that such facts as these, whereby all the parts of the whole existing Anglo-Saxon capacity and weight measures are not only connected with the pyramid, but are made to harmonize so exactly with each other, are "mere coincidences," and that they exist simply because they have been looked for? The class of facts to which these discoveries belong does not exist unless *intentionally* created by a scientific use of the intellect, and of the fact of such intention we may be as certain as of the correlative one that it requires an intellect to comprehend them.

The several tables of Anglo-Saxon weights which we have thus rectified at the pyramid will, therefore, stand as follows:—

RECTIFIED TROY WEIGHT.

24 grains	= 1 pennyweight	= 31$\frac{1}{4}$ ultimates.		
20 pennyweights	= 1 ounce	= 480	grains	= 625 ultimates.
12 ounces	= 1 pound	= 240	dwt.	= 5,760 grains = 7,500 ults.

Upon its surface the above table, save in the use of the additional ultimate, does not appear to differ from troy weight as given in any book of tables. This is because we preserve all the terms and the form of the old system. This is one of the chief merits in the value of our discovery, that, while by this system the weights now in use become absolutely perfected, no confusion can result in their adoption.

RECTIFIED APOTHECARY WEIGHT.

20 grains	= 1 scruple	= 26$\frac{1}{24}$ ultimates.				
3 scruples	= 1 dram	= 60	grains	= 78$\frac{1}{24}$ ultimates.		
8 drams	= 1 ounce	= 24	scruples	= 480	grains	= 625 ultimates.
12 ounces	= 1 pound	= 96	drams	= 288	scruples	= 5,760 grains = 7,500 ults.

Similar remarks are in order relative to the above table.

RECTIFIED AVOIRDUPOIS WEIGHT.

30 grains	= 1 dram	= 39$\frac{1}{16}$ ultimates.			
16 drams	= 1 ounce	= 480	grs.	= 625 ultimates.	
16 ounces	= 1 pound	= 256	drs.	= 7,680 grs.	= 10,000 ultimates.
28 pounds	= 1 quarter	= 448	oz.	= 7,168 drs.	= 215,040 grs. = 280,000 ults.

4 quarters = 1 cwt.	= 112 lbs. = 1,792 oz. = 28,672 drs. = 860,160 grains = 1,120,000 ultimates.	
20 cwt. = 1 ton	= 80 qrs. = 2,240 lbs. = 35,840 oz. = 573,440 drams = 17,203,200 grains = 22,400,000 ultimates.	

In the avoirdupois table the present form and skeleton are preserved, save that the effect of our discovery is to rectify the form in one particular; namely, 30 of our new grains equal 1 dram, while in the present statute-tables 27.34375 of the old Anglo-Saxon grains go to form the dram. The system thus clears the most important or commercial weight of its awkward fractional commencement, besides making its ounce, grain, and ultimate of the constant or unit value of all the other tables.

To complete the series of tables, we shall now add the following one of intermediate values derived from the standard pyramid pound of 10 unit ounces, and which we shall designate as :—

STANDARD WEIGHT MEASURE (RECTIFIED).

12 grains	= 1 scruple = 15⅝ ultimates.[1]	
4 scruples	= 1 dram, or minim = 48 grains = 62.5 ultimates.	
10 drams	4 drams = 1 table or large spoon 2½ large or tablespoons = 1 ounce	= 1 ounce = 40 scruples 480 grains = 625 ults.
10 ounces	4 ounces = 1 gill 2½ gills = 1 pound	= 1 pound = 100 drams = 400 scruples = 4,800 grains = 6,250 ultimates.
10 pounds	(4 pounds = 1 quart wt.) = 400 drams (2½ quart wt. = 1 stone) = 10 lbs., etc.	= 1 stone = 100 ounces = 1,000 drams.
10 stone	(4 stone = 1 sack wt.) = 40 lbs. = 400 ounces = 4,000 drams. (2½ sack wt. = 1 quintal) = 100 lbs., etc.	= 1 quintal = 100 lbs. = 1,000 oz. = 10,000 drams.
10 quintals	(4 quintals = 1 wey) = 40 stone = 400 lbs. = 4,000 oz. = 40,000 drams. (2½ wey = 1 thous. wt.) = 100 stone, etc.	= 1 thous. wt. = 100 st. = 1,000 lbs. = 10,000 oz. = 100,000 drams.
10 thous. wt.	(4 thous. wt. = 1 ton) = 40 quintals = 400 st. = 4,000 lb. = 40,000 oz., etc. (2½ tons = 1 bin) = 10 thous. weight = 100 quintals, etc.	= 1 bin = 100 quintals = 1,000 stone = 10,000 lbs = 100,000 ounces, etc.
4 bins	= 1 standard = 10 tons = 40 thous. wt. = 100 weys, etc.	

[1] The "ultimate" in these tables is of special advantage in calculation. Having played, however, its most important part as a factor in the process of rectification, it may, in fact, be entirely left out of the tables, and employed only as a means of subordinate intercourse, as it were, from one to the other. But as the system develops into its widening possibilities, this subdivision from its direct connection with the decimal notation will prove of great convenience.

In the above table, which is a new one, but in which, for facility of introduction, old and well-known Anglo-Saxon terms are employed, the most noticeable feature is its decimal continuity. Thus, commencing at the standard dram, this table may be read decimally as follows: —

10 drams	=	1 ounce.
10 ounces	=	1 pound.
10 pounds	=	1 stone.
10 stone	=	1 quintal.
10 quintals	=	1 thousand weight.
10 thousand weight	=	1 bin.
4 bins	=	1 standard.

Or, since it is doubly decimal, we have the following alternate system pervading it: —

10 spoonfuls	=	1 gill.
10 gills	=	1 quart.
10 quarts	=	1 sack.
10 sacks	=	1 wey.
10 weys	=	1 ton.
10 tons	=	1 standard.

Moreover, a careful study of the table of standard weight measure rectified will show the facility with which any and all of its various terms may be halved, quartered, and multifariously subdivided into common fractional parts. Thus, 1 pound equals $\frac{1}{4}$ of a quart; 5 ounces equal $\frac{1}{2}$ a pound; $\frac{1}{4}$ of a sack equals 1 stone, etc. The completeness with which this common fractional subdivision, so necessary in the daily use even of a decimal system, may be effected, will be best understood from the following: Since there are 4,800 grains in a standard pound, and since $4{,}800 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 3 \times 1$, it follows that the pound may be divided into $\frac{1}{2}$'s, $\frac{1}{3}$'s, $\frac{1}{4}$'s, $\frac{1}{5}$'s, $\frac{1}{6}$'s, $\frac{1}{8}$'s, $\frac{1}{10}$'s, $\frac{1}{12}$'s, $\frac{1}{15}$'s, $\frac{1}{16}$'s, $\frac{1}{20}$'s, $\frac{1}{24}$'s, $\frac{1}{25}$'s, $\frac{1}{30}$'s, $\frac{1}{32}$'s, $\frac{1}{40}$'s, $\frac{1}{48}$'s, $\frac{1}{50}$'s, $\frac{1}{60}$'s, $\frac{1}{64}$'s, $\frac{1}{75}$'s, $\frac{1}{80}$'s, $\frac{1}{96}$'s, $\frac{1}{100}$'s, $\frac{1}{120}$'s, $\frac{1}{240}$'s, etc., and hence that every term in the table may likewise be so subdivided without fractional remainders, etc.; hence, if a decimal table of weights and measures be actually necessary at the present state of science and commerce, we have one in the system now discovered whose introduction will involve no change of terms in Anglo-Saxondom, whose unit (the ounce) is that of troy, apothecary, and avoirdupois weights, and one, too, in which the decimal system is not only double, but which offers extraordinary facilities for the still more necessary employment of common fractions, etc.

In all of the foregoing tables, the ultimate, grain, ounce, and gill are CONSTANT, or have a "standard" value: the other subdivisions are the *particulars* of special arts, yet all of them are perfect and direct parts of the "unit ounce weight." The constant values are as follows: —

ANCIENT PYRAMID, ORIGINAL ANGLO-SAXON, AND RECTIFIED.

A "standard."	"ultimate" =	.00285 (exact) pyramid cubic inches of pure water at mean earth temperature, pressure, and hygroscopic state (pyramid 50°) $= \frac{\text{ounce}}{625} = \frac{\text{gill}}{2,500} = \frac{\text{coffer}}{25,000,000}$.
	"grain" =	1.3020833, etc. (i.e. $\frac{125}{96}$ or $1\frac{29}{96}$) grains = .0037109375 (exact) pyramid cubic inches pure water, pyramid 50°, temperature, barometer, and hygros. $= \frac{\text{ounce}}{480} = \frac{\text{gill}}{1,920} = \frac{\text{coffer}}{19,200,000}$.
	"ounce" =	480 grains = 625 ultimates = 1.78125 (exact) pyramid cubic inches of pure water at 50° pyramid, temperature, barometer, and hygroscope $= \frac{\text{gill}}{4} = \frac{\text{quarter}}{10,000} = \frac{\text{coffer}}{40,000}$.
	"gill" =	1,920 grains = 2,500 ultimates = 7.125 (exact) pyramid cubic inches of pure water at 50° pyramid, thermometer, barometer, and hygroscope $= \frac{\text{quarter}}{2,500} = \frac{\text{coffer}}{10,000}$.

It is noticeable, that, in every one of these tables, the standard ounce is ultimately divided into 480 grains. Thus, in troy, we have $24 \times 20 = 480$; in apothecary, $20 \times 3 \times 8 = 480$; in avoirdupois, $30 \times 16 = 480$; and, in the proposed standard, $12 \times 4 \times 10 = 480$. By means of these arrangements, every fractional part of this unit ounce that trade, art, or science can demand is thus specifically *named*. Thus, to mention but a few, the half-ounce = 8 drams avoirdupois = 4 drams apothecary = 10 pennyweights troy = 5 drams standard; $\frac{1}{3}$ ounce = 8 scruples apothecary; $\frac{1}{4}$ ounce = 10 scruples standard = 4 drams avoirdupois = 2 drams apothecary = 5 pennyweights troy; $\frac{1}{5}$ ounce = 4 pennyweights troy; $\frac{1}{6}$ ounce = 4 scruples apothecary; $\frac{1}{8}$ ounce = 1 dram apothecary; $\frac{1}{10}$ ounce = 2 pennyweights troy; $\frac{1}{12}$ ounce = 2 scruples apothecary; $\frac{1}{16}$ ounce = 1 dram avoirdupois; $\frac{1}{20}$ ounce = 1 pennyweight troy; $\frac{1}{24}$ ounce = 1 scruple apothecary; $\frac{1}{32}$ ounce = $\frac{1}{2}$ dram avoirdupois; $\frac{1}{36}$ ounce = $\frac{2}{3}$ scruple apothecary; $\frac{1}{40}$ ounce = $\frac{1}{2}$ pennyweight troy; $\frac{1}{48}$ ounce = $\frac{1}{2}$ scruple apothecary, etc., by suitable combinations.

All of the above tables are strictly Anglo-Saxon, and they are as strictly of Great-Pyramid origin. For all practical purposes, the subdivision of the coffer into 19,200,000 parts furnishes us with an elementary weight sufficiently small; and it is upon this fractional subdivision that our ancestors manifestly agreed when they established those subdivisions, or aliquot numbers, for the various pounds, — 4,800, 5,760, and 7,680, — which we have come to designate as "grains." This, however, exceeds the true pyramidal ultimate of weight, as we have shown, in the proportion of 250 : 192 = 125 : 96; so that the 19,200,000 grains, into which the coffer is thus resolved, are actually equal to 25,000,000 real ultimates of .00285 pyramid cubic inches capacity of pure water each. Such an ultimate is equal in weight to $1\frac{3}{5}$ centidrams of water (at .00178125 pyramid cubic inches per centidram), as given in the "Table of Pyramid Measure, extended Downward." (See p. 56.)

Both of these numbers, 19,200,000 and 25,000,000, expressive respectively of the number of grains and ultimates in the standard coffer, are remarkable for the facility with which they may be successively factored. This is a property of manifest and primary utility for any radical that shall be assumed as the basis of minute capacity or weight measures. Thus, eleven successive divisions of the number 19,200,000 by the constant factor 2 gives us the quantity 9,375, which by six more successive divisions by the factor 5 reduces to unity itself, i.e., to one grain; that is, $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 1 = 19{,}200{,}000$. Or, viewed from another stand-point, it is successively divisible by 10 seven times, the remainder in this case being 1.92 (exact) grains: $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 1.92 = 19{,}200{,}000$.

Now, $1.92 = \frac{192}{100}$, and in ultimates $= \frac{192}{100} \times \frac{125}{96} = 2.50$ ultimates, the cubic capacity of which is but .007125 (exact) pyramid cubic inches; i.e., the drop. The subdivision can be thus continued down to what is literally *the last drop in the coffer.*

Again, taking the number 25,000,000, and treating it in a similar way, we find, that, in spite of its complex relations to 19,200,000 (i.e., 125 : 96), it can be six times successively divided by 2, yielding the number 390,625 grains, which, by eight more successive divisions by 5, likewise reduces to unity. Thus, $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 1 = 25{,}000{,}000$. Or, factoring by 10 six times, we obtain the minute quantity of 25 ultimates; after which we may still factor once or twice more by 5. Thus, $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 5 \times 5 \times 1 = 25{,}000{,}000$; leaving respectively either 5 or but 1, and this one absolutely the last ultimate of all its fellows.

Now, as in the case of capacity measures, so, with even more appearance of necessity, we may continue the standard table of weight downward from the unit ounce and gill as follows:—

STANDARD WEIGHT MEASURE, EXTENDED DOWNWARD.

10 millidrams	4 millidrams = 1 atom. 2½ atoms = 1 centidram.	= 1 centidram.
10 centidrams	4 centidrams = 1 drop. 2½ drops = 1 decidram.	= 1 decidram.
10 decidrams	4 decidrams = 1 dose. 2½ doses = 1 dram.	= 1 standard dram.
10 stand. drams	4 stand. drams = 1 large spoon. 2½ large spoons = 1 ounce.	= 1 ounce.
4 ounces		= 1 gill.

It will be noticed that there runs through this table, as in the case of all the other standard ones, a subordinate decimal system of weights, which is as follows:—

10 atoms	= 1 drop.
10 drops	= 1 dose.
10 doses	= 1 large spoon.
10 large spoons	= 1 ounce.
4 ounces	= 1 gill.

The exact number of grains in a cubic inch of water (pyramid 50°, temperature and pressure and hygroscope) is expressed decimally by the repetend, 269.47368421052631578947368 + etc.; and the number of ultimates in such a cubic inch by another repetend, as follows: 350.877192982456140350 + etc.

The above numbers (though capable of exact use in calculation, on account of their repeating character) look formidable

But for all practical purposes they can safely be employed, as decimals, in the following forms: grains in 1 cubic inch = 269.4737 (—), and ultimates in cubic inch = 350.8772 (—). We shall return, however, to these two repetends in due time, when it will be shown that the beauty of the whole system concentrates in them, and when their consummate simplicity for the purposes of accurate subdivisions *into practical commercial weights of all descriptions for* USE UPON THE SCALES *will be revealed.*

In the mean while let us return to the coffer, which is now, and in this connection, to have for us an even higher signification than any upon which we have yet touched.

The best modern estimate of the mean density of the earth is about 5.7 times the weight of water. To determine this accurately has been one of the greatest problems of modern times. It has enlisted the ability of the greatest scientific men. Newton shrewdly guessed, without experiment, that it would be found between five and six times that of water.[1] Maskelyne followed with a series of unsatisfactory experiments making it 4.8 water, with an addition whose amount he could not determine. Capt. Ross Clark, R.E., determined upon 5.316. Sir George B. Airy, Dr. Whewell, and Richard Sheepshanks, working in the mines at Cornwall, met with failure in 1855. Later, however, Sir G. B. Airy, in a mine near Newcastle, was somewhat more successful, but arrived at "the unexpectedly large result of 6.565." Rev. John Mitchell next proposed a new mode of determination, which was later carried out by Cavendish with 5.45 as a result. Professor Reich of Saxony repeated the experiments of Cavendish, and deduced 5.44. Finally Francis Baily, after a most protracted series of investigations, determined this mean density to be 5.675 ± .0038. Even yet, however, modern science is not satisfied and certain, though it is well convinced that the quantity sought for is around the estimate made by Newton, 5.5, with a possible addition of from .1 to .3, in the positive direction, from the indications of the more careful experiments of Baily.

The Great Pyramid, however, pronounces emphatically for the figures we first quoted, or for a mean density of the earth of

[1] It was Newton, too, who also first made the remarkably close approximation to the length of the "sacred cubit," 24.88 ± British inches.

just 5.7 times that of water *in the king's chamber;* i.e., at mean temperature 50° pyramid, barometer 50° pyramid, and hygroscope 50°.

I shall not at present attempt to adduce the various incidental references found in the pyramid, and particularly in the neighborhood of the granite leaf and coffer, to this number 5.7, but, assuming it to be the quantity sought and intended to be employed in pyramid metrology, shall proceed at once to examine into the circumstances that will attend its introduction as a factor into our investigations.

We have already noticed, that by filling the coffer with water at 50° pyramid, barometer, thermometer, and hygroscope, it is converted from a capacity measure into one of weight. Such a cofferful will, of course, be 71,250 cubic inches of water. The question now arises, What will be its equivalent at the mean density of the earth? Dividing 71,250 by 5.7, to obtain an answer to our question, we obtain for the result the noticeably neat number 12,500 cubic inches. This, therefore, is the bulk of the earth's mean density material which the coffer indicates, and *which will at all events exactly balance a cofferful of water under standard circumstances.* This is a point that should be clearly appreciated before we proceed; namely, that, be the mean density of the earth 5.7 exactly or not, still, and nevertheless, 12,500 cubic inches of material thus (5.7) dense will exactly balance 71,250 cubic inches, or a theoretical cofferful of the unity (1.) material water in the Standard Observatory at Gizeh as well as elsewhere all over the earth amid the habitations of man, duly preserved at mean earth temperature, pressure, and hygroscopic state.

This fact established, we are now ready to examine our number 12,500 at length. It is first noticeable, that this number is exactly one-tenth *the cube of* 50, — a fact which connects it in a remarkable way with the *standard chamber,* already filled with architectural, barometric, hygroscopic, thermic, mathematical, geometric, and other references to this *marked* number, 50, in which the coffer was preserved at Gizeh.

Now since, as we have already seen, a coffer contains 2,500 pyramid avoirdupois pounds, each such pound is represented by 5 cubic inches of the mean density material; i.e., 12,500 ÷ 2,500

$= 5''$: hence it follows, that 50 such cubic inches will exactly balance 10 rectified avoirdupois or commercial pounds. To give a still better idea of this, it may be stated, that, if we could compress 285 cubic inches of water (10 pyramid avoirdupois pounds) into a space of 50 cubic inches, it would be at the mean density of the earth, or, at least, at standard pyramid density. For similar reasons, a standard or 10-ounce pound $= 3\frac{1}{8}$ cubic inches of this material, and a rectified troy or apothecary pound $= 3\frac{3}{4}$ cubic inches, etc. Now, the fact that 12,500 is $\frac{1}{10}$ of a cube of $50''$ of this standard material, leads to the idea that *the cube of* 50 *itself*, or 10 coffers of water, *is the actual pyramid standard for weight.*[1] Such a cube ($50'' \times 50'' \times 50''$), mean density, would contain exactly 25,000 commercial pounds, the cubit ($25 \times 1{,}000 = 25 \times 10^3$) character of which is noticeable. By referring now to our table of "Standard Weight Measure extended Upward," it will likewise be seen that this quantity has been designated as a standard, and is an average car-load, and that 10,000 such cubes $10^4 \times (50 \text{ mean density})^3 =$ an ark avoirdupois, or as many pyramid pounds avoirdupois as there are inches in the polar radius of the earth.

As in the case of capacity measures by means of this standard cube, the rectified system of weight measures may be extended upward, so as to comprehend the most extravagant demands that can be made upon it by a world at peace, and engaged in universal intercourse and international commerce. A scheme for such extended use is given below; and since a standard equals ten tons, or an average car-load, we have, —

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[1] It is also noticeable here, since $50''$ is equal to $2 \times 25''$, or to *two* cubits, that the cube of $50''$, or the standard cube, is itself, when expressed in cubits, peculiarly *tied*, as it were, to the octenary system of numeration. Thus $(50'')^3 = (2 \text{ cubits})^3 = 8$ cubic cubits. It is thus not only geometrically a cube, but arithmetically so, and this in a most important triple way; for taken in terms of the unit cube $[125{,}000 = (50'')^3]$, or in terms of the cubit cube $[(25'' \times 2)^3 = (2 \text{ cubits})^3 = 8$ cubic cubits$]$, or in its own integrity as a standard, it is equally expressive of this fundamental metrologic idea of cubicity so necessary in weight and capacity considerations, while at the same time it has an inherent connection with all linear measure through the inch, the cubit, and the axial reference. For important discussions upon the peculiar advantages of an octenary basis for metrology over a decimal one, and of the direct relation of the former to duodecimal arithmetic, see following articles: "Unification of Moneys, Weights, and Measures," International Review. "Extracts" therefrom, by Alfred B. Taylor, International Standard, July, 1883. "The Metric System in our Workshops," by Coleman Sellers, Journal of Franklin Institute, June, 1874. "Report by S. F. Gates," International Standard, March, 1883, etc.

STANDARD WEIGHT MEASURE, EXTENDED UPWARD.

10 tons	2½ tons	= 1 bin.	= 1 standard, or car-load.
	4 bins	= 1 standard.	
10 car-loads	2½ carloads	= 1 lighter.	= 1 barge.
	4 lighters	= 1 barge.	
10 barges	2½ barges	= 1 ship-load.	= 1 elevator.
	4 ship-loads	= 1 elevator.	
10 elevators	2½ elevators	= 1 wharf.	= 1 district.
	4 wharves	= 1 district	
10 districts	2½ districts	= 1 section.	= 1 ark, or harbor.
	4 sections	= 1 ark.	

In the above table, there runs the following subordinate decimal system:—

2½ tons	= 1 bin.
10 bins	= 1 lighter.
10 lighters	= 1 ship-load.
10 ship-loads	= 1 wharf.
10 wharves	= 1 section.
4 sections	= 1 ark, or harbor.

Now, it has been already shown how from the standard cube we may descend (unifying as we go the grandest system of weights — the Anglo-Saxon — that the world now possesses) to the very ultimates of troy, apothecary, avoirdupois, and standard weights. Let me, however, add, before dropping this explanation, that it is evident, if we consider the ultimate (=.00285 exact cubic inch pure water at mean temperature and pressure) a small enough subdivision to denominate here by a special name, nevertheless, shall science find it necessary to decimate the ultimate, grain, the ounce, or gill, it can do so to the very limit of numerical capacity by simply removing the decimal point one place farther to the left at every division by 10. In this case, if a name be necessary, let us denominate (following our now well-known monetary phraseology of dimes, cents, and mills) .000285 $= \frac{1}{10}$ of an ultimate a "dimultimate," .0000285 $= \frac{1}{100}$ of an ultimate a "centultimate," and .00000285 $= \frac{1}{1000}$ of an ultimate a "milultimate," etc. In a similar way we can have dimegrains, centigrains, and millegrains, and dimeounces, centiounces, and milleounces, etc.

One can hardly imagine even that the remotest science will ever be able to appreciate so small a quantity as a millionth of an ultimate. Nevertheless, as mathematics may write and employ in calculation a millionth even of a millionth (.000000000001) of an ultimate (.00285), this system admits of it in an exact number, = .00000000000000285, since the figure 5 in 285 is always final.

Let us now return to a consideration of the two repetends previously determined, as decimally expressive of the capacity of a cubic inch of water in grains and ultimates. These numbers are, —

1 cubic inch of water = 269.47368421052631578947368 + etc., grains.
" " " " " = 350.877192982456140350 + etc., ultimates.

Through the medium of the *mean density material*, these formidable numbers (nevertheless, perfectly *exact* even in their present condition) assume the form, not only of the most beautiful simplicity, but also of the most astonishing utility. Thus, —

12,500 cubic inches mean density = 19,200,000 grains, by weight; and
12,500 " " " " = 25,000,000 ultimates, by weight.

A cubic inch of mean density, therefore, contains exactly 1,536 grains = 2,000 ultimates = $\frac{1}{5}$ (pyramid) avoirdupois pound = $\frac{4}{15}$ (pyramid) troy and apothecary pounds = $\frac{8}{25}$ (pyramid) standard pounds, or = $3\frac{1}{5}$ unit ounces; i.e., pyramid and rectified avoirdupois, troy, apothecary, or standard. In this same connection we should notice that 10 times this coffer equivalent of mean density material, or 125,000 cubic inches mean density, or (50 inches mean density)^3 = 250,000,000 ultimates, or again the *semi-axial* number, and = 192,000,000 grains.

Let us now imagine ourselves in possession of a cubic inch of the mean density material, and examine the nature of its characteristic facilities as to numerical subdivision for the purpose of *weight-making*, or manufacture. The two numbers to be considered are 1,536 and 2,000; these, as above, being respectively the numbers of grains and ultimates in the cubic inch. Each of these numbers is capable of astonishingly complete resolution by factors. Thus, —

1,536 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3 × 1.
2,000 = 2 × 2 × 2 × 2 × 5 × 5 × 5 × 1.

By inspection of the above remarkable series of factors, *occurring so appropriately in the very place where they are most* *needed*, to wit, as components (in the known terms of grains and ultimates of all our Anglo-Saxon weight measures) of a convenient and definite cubical portion, i.e., of ONE UNIT INCH, *of mean earth, or standard material*, it is manifest that every conceivable fractional portion that may be demanded by art, science, or commerce can be realized, and this without the shadow of an error. The numbers 1,536 and 2,000, which thus express the value of the unit cube in terms of grains and ultimates, are in many other ways equally notable. Thus, 1,536, the more important of the two, as it subdivides the cube into the familiar grains, is *three times the cube of eight* ($3(8)^3 = 1{,}536$), — a fact of vast importance in *weight manufacture* when we remember the many desirable properties of 8 as a radix of metrological notation. Nor should it be overlooked here, that the number 8 is also itself a perfect cube. Thus, as more completely analyzed, $1{,}536 =$ *3 times, the cube of two cubed;* or $= 3((2)^3)^3$. Thus, too, from still another stand-point, the unit cube is made up of 192 elementary cubes of 2 on an edge, or of 8 grains capacity, the number 192 being itself resolvable by $2\times2\times2\times2\times2\times2\times3\times1$, or into halves, thirds, fourths, sixths, etc. Now, all of these cubic and even parts of the unit and elementary cubes will be found, upon inspection, to bear direct and important practical relation to the whole series of Anglo-Saxon weight measures as rectified, the advantage of which cannot fail to be seen by the scientific metrologist. We have already seen that the system grandly culminates in a still more important and standard cube; so that, from the elementary, through the unit, and up to the standard cube, and beyond, this unique system is not only upon its surface both decimal and common, but, when rightly understood, is even still more perfect in its complete compass of all the properties of the number 8.

Moreover, the number 1,536 is exactly divisible by every one of the digits except 7, and yields in this latter case a convenient repetend. In the possession of this latter property this number, therefore, occupies the same vantage-ground as the number 7,200, concerning the desirability of which for employment as the number of grains in the avoirdupois pound so much argument has hitherto been advanced. In this connection we may also note that the whole series of numbers express-ing the grains and ultimates in the several pounds as here rectified, possess exactly this same property. Thus, 4,800, 5,760, and 7,680, the number of grains per pound as rectified, and 6,250, 7,500, and 10,000, the corresponding number of ultimates, all yield to exact subdivision by the several digits, and, even in the case of subdivision by 7, yield convenient repetends. Similar properties are possessed by the number 2,000 (i.e., the ultimates in a cubic inch), which is equal to $2(10)^3$, and is divisible by the digits into commensurable parts. Hence the series of numbers here advocated as the only natural ones for a system of universal metrology, possess, besides those enjoyed by the number 7,200, other and far more important ones, which the latter and its connected numbers do not and can not enjoy. Finally, the factoring of the numbers belonging to the rectified series shows other important practical beauties, such as the following: $4,800 = (2)^4 \times 3 \times (10)^2$; $5,760 = 10(24)^2 = 2 \times 3 \times 5(2)^3$; $7,680 = 3 \times 5((2)^3)^3$; $6,250 = 10(25)^2 = 10((5)^2)^2 = 2(5)^5$; $7,500 = 3 \times 25 \times (10)^2 = 3(5)^4 \times (2)^2$; $10,000 = (10)^4 = (2)^4(5)^4$, etc.

In these numbers, in fact, as aliquot parts of this little unit cube, the grand metrological system of the Great Pyramid of Gizeh culminates. Crushed, as it were, into a toy, that would hardly fill an infant's hand, we have here for our contemplation the whole symbolism of this mysterious monument.

The unit cube! *Of what is it not significant?* Its edge, the linear unit, is the 1-25th of the sacred cubit, which in turn is the one 10-millionth of the polar-radius. By it the cube is directly related to all Anglo-Saxon, *and eternal,* linear measures whatsoever. By it also it is directly related to all chronology, to every motion of the earth. By it the pyramid itself — the mesocosm, as it were — is built anew. Yes, literally built anew; for 20,612 grains $\div$ 1,536 $= x =$ 13.16158203125 (exact), and 6,561 grains $\div$ 1,536 $= y =$ 4.271484375 (exact), and 20,612 : 6,561 :: $x : y$ or as $\pi : 1$ by the PARKER MODULUS! This is the hardest blow of all that the pyramid can give to modern science; for it is easier for a camel to go through the "needle's eye" than for the modern scholar to admit the circle could have been squared to such a degree of accuracy at this early day, or that the value of the π was ever better than a mechanical approximation, until the intellect of our vaunted days discovered the calculus.[1]

By the square of its edge, i.e., by one of its six small faces, the unit cube is related to all area; and by its six unfolded is revealed to our astonished eyes the very symbol of salvation unto all who dwell upon the surface of the earth.

By the volume of the cube, capacity itself is meted out. The 1-71,250th of the mysterious coffer, it measures food and drink, and is correlated through art and science with every thing on earth. Finally by its weight it consummates itself. The 1-125,000th of the perfect or standard cube, it is the earth itself in miniature, and leaves within the range of all metrology no unit lacking.

This is not rhapsody, nor poetry, nor the scientific, nor unscientific, use of the imagination: it is solid fact, and all concentred in a cubic inch, — the standard mean of every thing man needs for measure.

Into this narrow compass we have thus at last compressed the whole significance of the Great Pyramid! and, as it grew the less, its wonder only grew the more! Truly, then, can all men say, "This is indeed that wonder in the midst of Egypt, and that altar on its border," of which the prophet spake. It makes no compromise whatever with false science or its advocates. Without error itself, it stands as an enduring emblem of confusion unto all who undertake to balance its eternal TRUTH with "vanity and words."

"Who," then, "is this that darkeneth counsel with words

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[1] It is not my purpose, here, to enter into any such forbidden ground as the domain of this much vexed question. Parker's modulus, be it true or false, is by no means a weak attempt at its solution; and his ratio, 20,612 : 6,561 = 3.141594 + etc., arrived at by an entirely independent course of reasoning, is, at any rate, of an eminently practical character. It exceeds that of Playfair and Legendre by a little more than unity in the 6th decimal place, or is about 1-3,000,000th greater. Modern science, therefore, which is fully satisfied with the ratio $\pi = 3.1416$, for all practical purposes, can certainly find no fault, for similar purposes, with Mr. Parker's very simple common-fractional form $\frac{20612}{6561}$, which gives us a value of π *true, to within unity, two decimal places beyond!*

My only object in alluding to this subject here at all, is to point out the fact of this astonishing π ratio being present in the value of the unit cube. Since it is here, it must be likewise present in that of the corresponding unit sphere of 1,536 grains weight, whose linear dimensions, surface, solidity, etc., will therefore necessarily reflect similar numerical beauties throughout the region of what may be designated as spherical metrology; or weights, measures, and volumes of spherical forms.

without knowledge?" With Charles Latimer "I ask Mr. Proctor and President Barnard to cease their flippancy, and answer the following and the foregoing."

I challenge any astronomer or "metric philosopher" to refute the arguments contained therein. These Anglo-Saxon facts are true, or they are false. There is but *one* eternal truth, *one* justice among men, *one* perfect weight. If the secret of the pyramid, Israelitish, Anglo-Saxon, international, and earth-commensuric metrology, be not comprised within this little cubic inch of the earth's mean density material, then let it be disproved; but, if it be so, it is then a solemn fact to all who dwell upon the earth.

Why, then, we may ask, with Eliphaz the Temanite, "Should a wise man utter vain knowledge, and fill his belly with the east wind?" Why "Should he reason with unprofitable talk? or *with speeches wherewith he can do no good?*" It is said that "there is such divinity doth hedge a king," and, again, that an undevout astronomer is "mad." Where is the divinity that hedges an undevout astronomer? It is time that this hiding and belittling of the grandest facts should cease; and Mr. Proctor should be asked, "Where wert THOU when the foundations of the pyramid were laid?"

But let us return to the consideration of this wonderful cubic inch. Five of them make a pyramid avoirdupois pound, and a sixteenth thereof gives us the unit ounce. In other words, a stick of the mean density material five inches long, and one square inch in cross-section, assists us in passing from the unit cubic inch to the unit ounce weight, by means of a careful subdivision into 16ths.

But of what shall we manufacture this stick or span-length of the mean density material, if we are to employ these facts for practical purposes? An alloy of aluminum (specific gravity 2.6 $\mp$) and copper (specific gravity 8.8 $\pm$) may be made to answer, since the proportions of the alloy may be so adjusted as to easily bring a cubic inch of it to a density of 5.7 times that of water. Thus, as a basis of experiment, we have $\frac{1}{2}(8.8(\pm) + 2.6(\mp)) = 5.7$.

There are also many other alloys by means of which we can realize this standard density. But an examination of a table of specific gravities will reveal to us that one of our best-known substances — the one, in fact, whose Latin name is significant of weight itself, i.e., LEAD (or plumbum) — has a mean density (11.4) exactly double that of the mean density material! Thus: [(water = 1) × (5.7 = mean density material)] × 2 = 11.4 = lead.

Of course, in the manufacture of weights, this double density is an advantage; since it lessens by one-half the size of every weight. Perhaps, therefore, no better material can be obtained than this one, offered, as it were, by Nature herself. It is cheap as well as heavy, much heavier than iron (7.125); and, though not quite so hard, a trace of antimony will strengthen it; and a duly proportioned standard alloy (11.4), with lead as the basis, will satisfy all our requirements.

Taking, however, pure lead (11.4), as a substance within easy reach of every workman, and as one indicated by Nature herself, it will be noticed, that, in being twice as heavy as the mean or standard density material, its number of aliquot parts, i.e., grains and ultimates per cubic inch, become (1,536 × 2) = 3,072, and (2,000 × 2) = 4,000 respectively.[1] In other words, while the cubic capacity of the weight standard, $(50'')^3 =$ 125,000 cubic inches, may be allowed to remain the same, we will, if it be now filled with lead, have doubled its already exhaustive powers of aliquot subdivision.

The standard 50-inch cube of pure lead (density 11.4) weighs exactly 50,000 pyramid avoirdupois pounds, one-tenth of which = 5,000 pounds = (2 × 50 × 50) will balance a cofferful thereof.

For the finer weights used in troy and apothecary measures, heavier, and particularly harder, metals may be employed. So, too, the standard national sets of weights may be made of still better material. For these purposes the metals gold (19.26), platinum (21.5), and iridium hammered (24), may be alloyed so as to produce some convenient multiple of the mean density material, as, for instance, 3(5.7) = 17.1 and 4(5.7) = 22.8, etc.

[1] Of course the actual cubic volume of lead necessary to balance a cofferful of water is ½ (12,500) = 6,250 cubic inches lead, each cubic inch of which weighs two of those of the 5.7 material. It is also noticeable in this connection, that there are the same number of cubic inches of lead (at 11.4) in a ton or cofferweight, as there are grains in the 10-ounce pound of pyramid standard weight; i.e., 6,250: and similar numeric repetitions will be noticed running all through this beautiful system.

Such an employment of specific gravity, to wit, for the purpose of obtaining exact cubical volumes, which shall, in turn, be related to each other in the same numerical sequences as are the terms of our tables of metrology themselves, and such an employment, too, as shall result in equal facility for binary, octenary, or decimal usage, is the concentrated "Wisdom of the Great Pyramid." It is the lesson written in its wonderful proportions by the mighty intellect that planned it for the modern scholar. Its scheme is universal; and not until the science of this latter day shall willingly repair to Gizeh, and study in its grateful shades, and square and right itself by its hoary cosmic truths, can modern science found itself upon the everlasting rock of stability, and rear itself a pyramid eternal as the earth itself.

These cosmic truths flash out and round about this mystic monument from every point of view. For instance, it is built of limestone on a limestone hill. Why, may we ask? May it not be because of the following significant facts? The mean specific gravity of marble or limestone is put down by Trautwine as 2.65 to 2.85 times that of water. The latter is a familiar number to those who have followed us carefully in our discoveries. It is one-half of 5.7, the mean density of the earth. Furthermore, it is, as an abstract number, one-tenth of 28.5, the number which expresses the cubic capacity of a rectified avoirdupois pound in water. Hence 10 cubic inches of 2.85 limestone or marble is a commercial pound. Moreover, since there are 1,728 cubic inches in a cubic foot, a cubic foot of such limestone will weigh exactly one-tenth as many, or 172.8 avoirdupois, pounds. Surely such facts as these are more than remarkable: they are calculated to actually stagger modern science in their endless suggestions of practical employment in the arts and trades and intellectual pursuits of man.[1] A cubic foot of the mean density material weighs

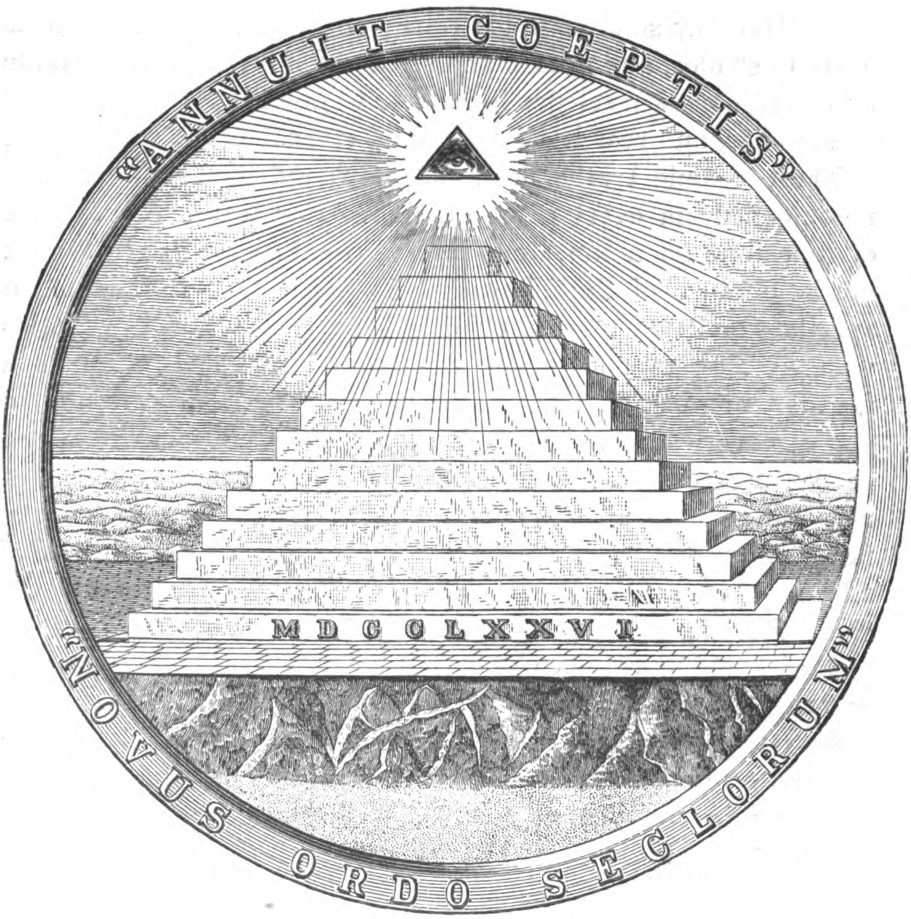
[1] I do not wish to be understood as claiming here that the Great Pyramid is actually built out of limestone of this particular (2.85) specific gravity, for such does not appear to be the fact. I simply wish to note, in passing, what may, perhaps, be at least an indirect reference to the subject, and to call attention to the possibility of such facts being not only of practical value, but here and there implied in the pyramid's significance. The specific gravities of some of our best and most useful building materials, — granite, limestone, marble, basalt, porphyry, slate, etc., — all arrange themselves around this particular degree; and future study of that all-important topic may result in discoveries of great value to commerce and science.

THE REVERSE

TO THE

GREAT SEAL OF THE UNITED STATES

OF AMERICA.



" And the rain descended, and the floods came, and the winds blew, and beat upon that house; and it fell not: for it was founded upon a rock." — MATT. vii. 25.

twice as much as one of 2.85 material, or 345.6 pounds. Hence one of water weighs in avoirdupois pounds $345.6 \div 5.7 = 60\frac{12}{19}$ (exact) pounds, or decimally $= 60.\dot{6}3152\dot{6}+$, etc., pounds, or for "practical purposes" $= 60\frac{5}{8}$ avoirdupois pounds. From this fact we learn that a cubic foot of any substance whatsoever (say specific gravity $= x$) must contain $60\frac{12}{19}\,x$ avoirdupois pounds, or $96\frac{48}{95}\,x$ standard pounds, etc. Thus, through the unit density, water (1), the mean density of our planet (5.7), that of certain grades of the well-known building material, limestone (2.85), and other geometric, numerical, specific gravity, and similar references, too numerous to mention, does the study of this building reach upward from our planet, and beyond to others, and from the grander numbers of the entire solar system outward farther yet unto the very boundaries of the universe itself, in search of loftier truths.

There was a day when so-called astronomers arrogantly looked down upon a certain class of numerical laws which the laity or non-schoolmen had found to exist among planetary distances, rotations, revolutions, etc. That day has passed away; and we now find an opinion constantly growing stronger among the leading scholars of this branch of science, that there must and does actually exist some universal law, by whose complete resolution into its appropriate terms and functions, an equation, as it were, of the solar system will result. Theoretically such an equation certainly does exist; and from the many simple relations already known to exist between densities, diameters, rates, and distances, we can be assured that it will be ultimately found to be by no means a complex one. The first step towards a true universal astronomy is the thorough understanding of all the "elements" of our own orb. It is the science of METROLOGY that teaches this, — a science, that, in view of the outlook afforded from so lofty an elevation as the Great Pyramid, we have already seen fit to define as "The Science of Round Numbers, Just Measures, and of Perfect Weights." Once awakened to the certain possibilities of such a scheme as the one the pyramid monumentalizes, man can, with ever deepening enthusiasm, look into Nature and her beauteous laws, and doubt not that the victory will be accorded to his intellect assisted from on high. We doubt not that our opinions here expressed will meet with bitter criticism. We challenge it indeed, for it will lead us to still higher and yet more impregnable positions. There are those who would presume to criticise the proportions of the throne of God, as rashly as they do the measures of his footstool.

But let us return to a consideration of the facilities afforded by this system for the practical manufacture of just weights and measures.

In all of the cases we have noticed above, the real object was, to obtain accurate sets of weights *for comparison* rather than for actual use. For this latter purpose, iron, as heretofore, will undoubtedly continue to be the most suitable. This is on account of its great hardness, and the consequent permanence of its stamped values.

Now, Nature comes to our assistance at this point in quite as marked a way as she did with the material lead. Trautwine puts the specific gravity of cast-iron at from 6.9 to 7.4, and tabulates the average at 7.15. This quantity does not appear to have any direct or convenient relation to 5.7, or to that of our mean density material, but it does *have a remarkably close one to the cubic capacity of the coffer itself;* i.e., to 71,250 cubic inches. In fact, it is perfectly within the powers of easy art to obtain an iron whose specific gravity shall always be exactly 7.125 times that of water, and of which, therefore, 10,000 cubic inches, $= 10 \times (10'')^3$ or to ten cubes of ten inches on an edge, will exactly balance the coffer. A cubic inch of such iron will weigh exactly one-fourth of a pyramid avoirdupois pound, and will therefore contain 1,920 aliquot grains and 2,500 ultimates respectively. Hence four such cubic inches weigh a pyramid avoirdupois pound; three of them, a pyramid troy or apothecary pound; and two and one-half of them, a pyramid standard pound. A ten-inch cube of such iron will weigh exactly 250 pyramid or rectified Anglo-Saxon pounds; and as in a standard cube $= (50'')^3$ there are 125 ($= (5)^3$) such ten-inch cubes, the standard cube of iron at 7.125 will weigh 31,250 pyramid avoirdupois pounds.

Now, the slightest investigation into the properties of the numbers involved in this rectified Anglo-Saxon system of Metrology, develops beauties in every direction; and their very existence establishes the supreme dignity of the treasure-house thus opened. Let us take, for instance, this last number, 31,250, pyramid or rectified Anglo-Saxon avoirdupois pounds, expressive of the weight of a standard cube (or 125,000 cubic inches) of iron.

The number itself is $10(5)^5$. Now, the fluid measures of ancient Israel (of divine origin, let it be remarked) were as follows: —

1 omer		= 178.125	(exact) cu. in. water, mean temp., press., and hygro.
10 omers	= 1 bath	= 1,781.25	" " " " " " "
10 baths	= 1 homer	= 17,812.5	" " " " " " "
4 homers	= 1 laver	= 71,250	" " " " " " "
50 lavers	= 1 sea	= 3,562,500	" " " " " " "
50 seas		= 178,125,000	" " " " " " "

Dividing now the cubic capacity of each of the above measures by 5.7, to reduce it from water to that of the mean density material, we obtain the several familiar numbers 31.25, 312.5, 3,125, 12,500, 625,000, and 31,250,000 cubic inches of mean density material. These numbers are respectively equal to $\frac{5^5}{100}$, $\frac{5^5}{10}$, 5^5, $4(5)^5$, $10{,}000(5)^5$.[1]

But the dry measure of Israel corresponded exactly with its fluid measure. Thus the unit of the former was the gomer of 178.125 cubic inches capacity of pure water at mean earth temperature and pressure and hygroscopic state, which, for testing-purposes, would have been balanced by exactly 31.25 cubic inches of mean density material. This dry measure was as follows: —

1 gomer		= 178.125	cu. in. water	= 31.25	m'n dens. material, to bal.
10 gomers	= 1 ephah	= 1,781.25	" "	= 312.5	" " "
10 ephahs	= 1 cor	= 17,812.5	" "	= 3,125	" " "
4 cors	= 1 ark	= 71,250	" "	= 12,500	" " "

The dry measure proper terminated at this point; but there are the very best of mathematical reasons for believing that the

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[1] I simply wish to note here, that the volume of the "Holy of holies" was 125,000,000 cubic inches, and, if filled with water, would have been balanced by an amount of mean density material equal to $125{,}000{,}000 \div 5.7 = 21{,}929{,}824.5614035087719$, etc., a repetend which seems to have no earthly connection with the sequences of figures, 178,125, 71,250, 31,250, which we have noticed in the above tabulation. That it *does*, however, we shall see later on.

actual contents of the great ark of Noah were based upon a value equal to 100,000 arks of the covenant, each of which held 71,250 cubic inches of water, or, as we have just seen, would have each been balanced by 12,500 cubic inches of mean density material.[1]

Now, there can be no doubt but that the whole religion of Israel was one concentred about the grand principle of the perfect equality of men in their dealings each with each. "*A just and perfect measure shalt thou have,*" and "*With what measure thou metest, it shall be measured unto thee again,*" were the underlying tenets of the whole Hebrew polity.

These are facts, and not fancies. The whole furniture of the temple, and of the earlier tabernacle, was constructed upon a consummate system of weights and measures, and finally culminated in a chamber called the "Holy of holies," whose specially peculiar construction has ever been a vast mystery. In this sacred chamber was the ark, — an imperishable standard of weight and capacity, — and in the latter were carefully stored a pot, or omer, of manna, the staple and symbol of the dual life of man, — the rod of Aaron (a cubit in length), and the decalogue, or measure and standard of the perfect life; while outside and beside it were other measures of equal import. Now, in all this careful arrangement, "by number, weight, and measure," there is design, of course, and in that design a system which our investigations clearly prove to be the embodiment of earth-commensuric metrology.

I shall now ask you to notice its wonderful culmination in the sacred cube called the Holy of holies. The dimensions of this cubical chamber were 20 cubits $\times$ 20 cubits $\times$ 20 cubits, or, as the sacred cubit was 25″, were 500″ $\times$ 500″ $\times$ 500″.

125,000,000 cubic inches was therefore its volume. Now, at the very first glance, this number is a familiar one. It is 1,000 standard cubes, 1,000 itself being also a perfect cube $= (10)^3$.

In our examination of the fluid measures of Israel we have just seen that the "sea" of 50 lavers contained 3,562,500 cubic inches of pure water at mean temperature and pressure,

[1] I refer those who desire to examine these reasons for themselves, to the Great Pyramid of Egypt, by Philo Israel, W. H. Guest, 20 Warren Lane, London, England.

or exactly 2,000,000 unit ounces (i.e., rectified Anglo-Saxon and pyramid ounces at 1.78125 cubic inches of water per ounce), and that 50 seas contained 178,125,000 such cubic inches, or 100,000,000 such unit ounces. We have also noted that the dry measure proper terminated with ark of the covenant, or laver volume of 71,250 cubic inches of water. Considered as a part of this intricate system, the volume of the cubic Holy of holies, 125,000,000 cubic inches capacity of pure water, seems, as we have before noted, to stand alone. But, by its very isolation, it is a marked number.

A more careful scrutiny shows that it is equal to 1,000 lavers, or "arks of the covenant," divided by the standard density, 5.7, — a fact which serves to bring this latter number (5.7) intensely to the front, and one which is still further marked when we note that the ratio between the ark of Noah (= 7,125,000,000 cubic inches, or 100,000 arks of the covenant) and the "sacred cube," or Holy of holies, is ten times this number, or as 57 : 1.

Thus we find continually repeated in the metrology of the Great Pyramid, in that of ancient Israel, and finally in that of the modern Anglo-Saxon, as now rectified, these important sequences of figures, 17,812.5, 71,250, 31,250, 125,000, 5.7, with whose unique fitness to the constitution of nature as a fact, we have actually forced and demonstrated our acquaintance. The first one is the cubic contents of a rectified quarter, the 10,000[dth] of which gives us the unit ounce; the second is the coffer or ton, a 10,000[dth] of which gives us the unit gill; the third is a standard cube, — weight of iron in true avoirdupois pounds; the fourth, the standard cube volume of mean density material in earth-commensuric cubic inches; and the last, the specific gravity of the earth itself, or the standard density by means of which, at mean temperature, pressure, etc. (50°), they are all so intimately correlated into a universal system of metrology.

Moreover, their relation, in the abstract, to the number 50 — a number so peculiar to the king's chamber in the pyramid, to the Holy of holies in the temple, and to the standard cube in the rectified Anglo-Saxon system — and to 25, the cubit number, is by no means to be slighted

But it is very difficult to convey any adequate idea of the consummate numerical beauty of this system of cubes, so intimately related to the Anglo-Saxon system of metrology as rectified, or of its vastly practical character. Thus, the grain being the element, 8 grains $= (2)^3$ grains, gives us an elementary or primary cube, which is $\frac{1}{192}$ of the cubic inch. The next cube we meet, and which may be called the secondary cube, is that of $(8)^3 = 512$ grains. The unit cube of 1,536 grains consists of three such secondary cubes, and the standard cube of $(50'')^3$ = 125,000 unit cubes. Again, —

1 grain	= .00065104166666, etc.	= $\frac{1}{1536}$ cu. in. of mean density material.		
1 elementary cube	= .005208333, etc.	= $\frac{8}{1536}$ " " " " "		
1 secondary cube	= .33333333, etc.	= $\frac{1}{3}$ " " " " "		
1 unit cube	= 1	= 1 " " "	=	$(1'')^3$
1 standard cube	= { 125,000 8 cubic cubits }	= 125,000 " "	=	{ $(50'')^3$ $(8 \text{ cu.})^3$ }
1 sacred cube	= { 125,000,000 1,000 × 8 cub. cubits }	= 125,000,000 " "	=	{ $(500'')^3$ $10^3 \times 8^3$ cu. }

Now, it is manifest, that, in terms of the grain, the primary and secondary quantities of mean density material are perfect cubes, and that the most important or unit, standard, and sacred quantities, are likewise so in terms of cubic inches; the latter being so in cubic cubits also. Hence, it is also evident, that, in one form or the other of their legitimate expression (i.e., either in grains, in cubic inches, or cubits), all of them may be put into cubic form; and hence that all of their derivatives, and their infinite combinations and multiples, may likewise be considered in a cubical character for the purposes of art, science, or trade, whenever such a consideration shall be desirable to facilitate calculation or practical wants. In fact, the field opened by these newly discovered elementary principles in metrology is broad enough to promise an everlasting harvest to intellectual man.

In terms of rectified Anglo-Saxon weights and measures, these Israelitish ones were as follows: 1 laver = 1 tun, or 4 hogsheads; 1 ark = 1 ton, or 2,500 avoirdupois pounds; 1 homer = 1 cor = 1 hogshead, or $62\frac{1}{2}$ gallons, at 285 cubic inches pure water at mean temperature and pressure; 1 bath = 1 ephah = $6\frac{1}{4}$ gallons; and 1 omer = 1 gomer = $2\frac{1}{2}$ quarts.

They are thus important elements in the grand and earth-commensuric system, and this is exactly what we might have expected; for if the system we have discovered be, as it undoubtedly is, the universal and only scientific one, and if the one of ancient Israel was God-given, the latter could not but have been also a part of the true system.

Let us now return for a moment to the standard cube of iron, whose weight, as we have seen, was 31,250 avoirdupois pounds. It consists of 125 subordinate cubes, each 10″ on an edge, and weighing respectively 250 pounds. Arranging these so as to form a continuous block of iron $(10'')^2$ in cross-section, the block will be 1,250″ long; and $\frac{2}{25}$ thereof, or 100 inches, will weigh 2,500 avoirdupois pounds, and balance a coffer or avoirdupois ton of pure water under standard circumstances, etc., for all avoirdupois weights; 25″ will balance the "thousand" pounds of standard weight, etc. In other words, a 1″ slab cut therefrom in cross-section will contain $(1'' \times 10'' \times 10'') = 100$ cubic inches, and therefore weigh 25 avoirdupois pounds $= 33\frac{1}{3}$ pounds troy and apothecary $= 40$ pounds standard weight, etc. So, too, all superior and inferior weights, in each of the several Anglo-Saxon, Israelitish, and Pyramid systems, — or, in general terms, in the earth-commensuric system, — will find accurate cubical expression therein.

But we need not go into this subject at greater length. Enough has surely now been adduced to establish beyond cavil the superiority, at every point, of the Anglo-Saxon measures, rectified at the pyramid and by the Bible, over the metric system.

I challenge the advocates of the metric system to bring forward their case, and to show that it can be put upon the same, or upon a corresponding, basis, as that on which I have demonstrated that the Anglo-Saxon one properly belongs.

I challenge them to produce any decimal part of their metre by means of which they can reach upward to astronomic distances, and downward into the ordinary affairs of men who halve and quarter by instinct.

I challenge them to show that the square of such a part shall be any natural superficial unit whatsoever, or that its cube shall relate itself to the volumes of nature or of the solar system

And finally, I then challenge them to show that such a cube when filled with water, a unity material to which men most refer; or with cereals, which they measure most; or the earth density material, as the manifest standard of weight; or with lead and iron, out of which man naturally makes his commercial weights; or with marble, as an enduring material for building; or with other things of similar importance to man, — shall in all these cases, or in any of them, possess even coincidences that shall have any systematic character whatever, and be related to the natural constitution of things, and therefore to each other, in grand round numbers. In other words, I challenge them to show that the metric system is the greatest common divisor, or any common divisor, or a least multiple, or any simple multiple, of the eternal constitution of things as existing around about us under measure, capacity, and weight.

I maintain, that from the very nature of measure, capacity, weight, etc., they are, and must be, directly related to each other, and to the earth itself, in some one system that shall be expressive, at its lowest terms, of all the elements of earth-commensurability. There cannot be two such systems, and there can be but one unit to the true system. This one true system, I maintain, is based upon the 10-millionth of the polar radius as the standard of length, upon its 25th as the unit of length, upon the square of this as the unit of area, upon its cube as the unit of volume, upon that volume filled with mean density material (5.7) as the unit cube, and upon that volume filled with the unity material as a basis of weight and capacity measures (1.78125 cubic inches = 1 ounce) in which unity shall be pure water (1) at mean earth temperature, pressure, and hygroscopic state. I claim, that by the aliquot subdivision of this unit cube of mean density material into 1,536 aliquots called "grains," and into 2,000 others called "ultimates" (from whence the unit ounces of capacity and of weight result), the entire Anglo-Saxon system becomes, not only harmonious, but rectified to nature as constituted in things men weigh and measure; that it realizes the pyramid metrology in its perfect scheme, and is throughout completely earth-commensuric.

I maintain that this, the true system, was not evolved by man, save as he found it already existing, like any other law in nature. I maintain, moreover, that man *per se* cannot create a new one upon arbitrary principles, and even hope to have it survive, except by the exercise of constant tyranny. I point to the experience of four thousand historic years of men constituted like ourselves for carrying on the affairs of ordinary daily life, and say that the universal testimony has been in favor of such systems of metrology as were primarily based upon aliquot parts not decimal; that decimal subdivisions, while important for certain branches of science as to the handling of particular units, etc., are not so for men at large; that, nevertheless, the natural system in its entirety is the most truly decimal system that can be imagined, and that to it all Anglo-Saxon measures are related.

I point to the experience of the past century, and claim that the French metric system itself, in this particular, is gradually changing its nature. It has not a single subdivision that men who are forced to use it do not halve and quarter. Look, too, at our own neat system of money. Though purely decimal, we halve and quarter far more familiarly than we decimate. Take away its dime, and what remains of its decimal character?

In another century, *if it be in existence still*, I predict that the metric system will have been forced to adopt common-fractional aliquots as at first a subordinate part of their metrology; that they will next have to name them; and that, if left to the people (as in modern times it must be) and to time, this system, from the nature of eternal things, will square and right itself without assistance *by the earth-commensuric cubic inch.*

What, then, could be more grandly earth-commensuric than the strange coffer in which the inch, the pint, and the ounce,— length, capacity, and weight,—temperature, pressure, hygroscopic state, and density,—bread itself and water,—iron, lead, and all things else,—are thus all joined in a most intimately harmonious reference, not only to the earth on which we live, but to our solar system and the universe itself?

The French metric system actually blasphemes the science of "Universal Metrology" in the light of such a system. It was enunciated in the era and the hot-bed of an awful atheism. Its falsely calculated metre was based upon the erroneous assumption, that all meridian quadrants of the earth were of equal length. We now know that probably no two of them are equal. It violated their first modern geometric principle in ever taking a curve for the standard of straight linear measure. It violated the very resolutions of its own Academy of Paris in so doing; for that academy has resolved, abiding by it yet, that a curved line cannot be squared or rectified, and that an infinite difference exists between the element of a diameter (straight) and that of a meridian (curved). If this be so, then it is ridiculous to base a scientific system upon what cannot be righted; the affairs of daily life being squared rather than curved.

The French system never dreamed of earth-commensuric capacity and weight standards. This is so because there can be no reason given why any particular cube filled with water shall be as (i.e., *just* as) earth-commensuric by weight as it is by volume, and by length and area as it is by both. The cubic inch (5.7) alone does all these things, and is alone the proper unit. Nor is it scientific to establish as a standard pressure, that which nature everywhere abhors (the vacuum), and for temperature that point (maximum density) around about which its very standard (water) most rapidly alters its condition. For the very reason (now so well approved) that astronomers go to the North star when off the meridian, and moving slowly in maximum elongation, and avoid the star when on the meridian, and in rapid motion, for the determination of the cardinal lines, so, in due regard to the eternal fitness of things, philosophy should, and science and common Anglo-Saxon sense do, veto the standard temperature on which the metric system is based.

The only advantage the French metric system possesses is its decimal notation. But the decimal system did not originate in France; it is older than tradition, and is personal to the human being; it is as readily applied to any other system; it is more ancient than the pyramid itself, and is built into it from the foundation up.

In our public schools this false, unscientific metric monstrosity is taught, to the utter confusion of all true metrological ideas. It is full time that we should banish it. It is not yet a century old. In its arrogant beginnings, it even sought "to change times and seasons." It made weeks of ten days, and contemplated years of ten months, and numerous other follies, which, briefly bolstered up by the guillotine, have each and all died violent deaths.

So long as the moon continues to lunate sidereally in periods of 27 days, 7 hours, 43′, 3″, 47‴, 20⁗, so long will men divide their time by weeks; and if tyranny divides time decimally, which is contrary to the nature of things, the moon herself will square it by the perfect cycle into weeks of seven days.

What is now known as the metric system is but the shadow of its former dimensions. Let us not allow it to live upon our soil to celebrate its first centennial. Let not an English-speaking, English-thinking, English-working, hard-handed Anglo-Saxon people forget their own traditions any longer. But let them listen to the silent teachings of that great monument upon the ancient Nile, from whence, mysteriously indeed, but nevertheless somehow, and in due time to be made clear, their own grand system is derived. Even the permissive use of the metric system is a blot upon our statute-books. There is no need of such a resolution. If men want to use an evil system, they will do it anyway. It is wrong, however, to give such a system any encouragement whatever, certainly short-sighted to recognize it at the nation's capital. But it is far worse to suffer it to be introduced into governmental acts and records. It is making, however, no headway among the people. For this we thank Almighty God. Out of 60,000,000 now among us, about 500,000 (!) — and I question even this as a *very* large over-estimate — scientific men have learned and use it, because *their foreign books are written in it.* The rest of our millions buy and sell their food and raiment in the good old Anglo-Saxon way.

Let the scientific read and use it: it is as much a part of their right and duty as to read and use the foreign languages themselves. They are our servants, and should translate these books into Anglo-Saxon weights, as well as Anglo-Saxon words. Shall the millions learn the metric system because a few thousands find their foreign theories written in metric hieroglyphics? As well might they demand that the Anglo-Saxon tongue itself should be declared dead and inarticulate, and literally bring about us Babel's own confusion, till the nation, stricken dumb, shall learn another speech.

The day of an international and universally earth-commensuric metrology is fast approaching. It was foreseen of old, from out the very shadow of that "sign and wonder" in the land of Ham; and thither to the future gate of universal commerce, man will surely yet return a willing pupil.

Why, then, shall we not rather hasten it by setting our own mansion in order, so that against the records of our own great generation may be also written, "At this time men rebuilt the 'Wonder of the World,' and universal progress dawned upon the earth"?

V.

RECAPITULATION.

"Let us hear the conclusion of the matter."

Principles upon which the Rectified System is based. — Its Primary Unit. — Standard Physical Conditions. — Special Standards. — Units and Primaries. — Constants.

(1) THERE must be some one system of metrology which best suits the whole constitution of all nature. There cannot be two "best" systems, nor can there be two equally good ones; for there can be but one least common multiple, or but one greatest common divisor, of any series of numbers, circumstances, or things.

(2) Such a system, when found, will be *ipso facto* the universal system, and therefore will be the only one fit to be international as such, in a day of general commerce, civilization, and freedom. Whatever that system be, it will be recognized instinctively by man as the Metrological Law of Nature; and, until it is discovered, man's metrology cannot but be a mere temporary expedient.

(3) To such a system, there can be but one primary unit.

(4) The aliquot subdivisions of this primary unit must not only suit the requirements of man's every-day life, and respond to his experience, but must square and cube themselves with geometrical magnitude, harmonize with the elementary constitution of things as expressed in specific gravity, and comprehend to the fullest degree the properties of number in the abstract.

(5) The system, therefore, must be both decimal and common-fractional, be tangent to nature in its elementary dimensions, its elements of capacity, and of weight, and, from all physical stand-points whence metrology is viewed, must rest upon natural standards.

(6) The standard circumstances of such a system should be the mean (each for its kind) of all the earth. Because — (1) It is the most natural. (2) The mean is most easily preserved. (3) It is as a fact more habitually used and preserved. (4) Man in perfect health and in full development, as the highest exponent of nature, lives, moves, and has his being, best at the mean of all terrestrial circumstances. (5) Extremes are always to be avoided, particularly in founding the bases of observation. (6) Variable circumstances change less in quantity, and less in rapidity (i.e., degree), around the mean.

(7) To be able to survive in the future day of universal science and commerce, — a day very near at hand, — an international system of metrology must satisfy all of the foregoing requisites. For man, as intellectually constituted, will not retain an unsound system; and any system that is merely tolerated, and which has apparent and inherent defects, must certainly go under.

(8) We are at the dawn of this new era. The Anglo-Saxon race transacts the business of the earth, and does it in its own time-honored system of metrology. The French metric system has apparent and inherent defects, and is used only by the decided minority of the civilized world, and by a minority destined to be outstripped in the struggle for existence. These defects are radical to the metric system; and, as such, they doom that system. It is, therefore, useless to expend time, thought, or money upon it. The Anglo-Saxon system, however, is already known to all the world, — even to the uncivilized thereof: it is already so nearly earth-commensuric, as to need but a slight rectification to enable it to embody nature's own law of metrology. This can and has been demonstrated. It therefore behooves the Anglo-Saxon race to "preserve the ancient landmarks," to square them and to right them, and to found them upon an everlasting basis; to set their own house in order, and to transmit to posterity a system, already international, and one that hereafter, to eternity, shall need no change. No amount of rectification can right the metric system, however; for it is founded upon unsound principles, and the principles themselves must be abandoned. It is not only non-lineal at the outset, but makes no pretensions to take advantage of

any of the properties of numbers other than the decimal ones, nor of specific gravity as regulating standard volumes of standard substances at standard circumstances. Its *rationale*, in fact, is not *harmonious* to nature in such a way as to express any grand law of metrology as a part of the constitution of things; and, failing here, it fails *in toto*, and *must* be abandoned.

(9) Let us now briefly reiterate the principles upon which we have worked in the foregoing system, with a view of fulfilling all the necessary requisites above enumerated. We have taken a cube of fifty linear units on an edge, and filled it with a material whose mean density is 5.7 at mean temperature and pressure. Then, since it contains 125,000 cubic units (inches) of such material, it must balance 712,500 cubic inches (units) of pure water under similar circumstances; and we have declared that such a weight shall consist of 192,000,000 grains, or 250,000,000 ultimates. Now, if the linear unit shall vary so as to make our present authorized inch either + or — to any small degree, it shall not alter the arithmetical fact that we will still use fifty of such new units as the edge of our standard cube. We shall maintain that specific gravity 5.7 is the proper standard density in terms of pure water at whatever may be mean temperature, pressure, and hygroscopic state of the earth (I now believe them to be 68° Fah., and 30″ barometer, and $\frac{1}{5}$ saturation), and we shall have the same number of grains and ultimates in the standard and unit cubes as we have above set forth. Having thus fixed our standard cube, and taken as our unit cube one of 1,536 such grains, or one 125-thousandth of the standard, we then establish the fact, that as our ultimate will occupy .00285 cubic inch of pure water at standard temperature and pressure, and as our ounce shall contain 625 ultimates, therefore our unit ounce weight shall balance 1.78125 cubic inches of pure water at standard temperature and pressure, or .3125 cubic inches of 5.7 material under the same circumstances; and by this unit ounce weight we intend to rectify all Anglo-Saxon weights whatsoever. We also declare that the space occupied by this unit ounce weight, when in pure water at standard conditions, — to wit, 1.78125 cubic inches, — shall be, and is, the unit ounce of measure or capacity.

By and with these weights and capacities, and their multiples, ranging upward and downward over the whole field of weight and capacity, we propose to rectify all present Anglo-Saxon measures, and to use the same in the manufacture of weights and measures.[1]

THE PRIMARY UNIT.

This is ONE CUBIC INCH OF MATERIAL, 5.7 TIMES AS DENSE AS WATER.

Its edge is the linear unit, or one inch $(1'')$. Its face is the superficial unit, or one square inch $(1'')^2$. Its volume is the cubical unit, or one cubic inch $(1'')^3$, and measures all geometrical volumes. 1.78125 : this cubical unit is the capacity ounce, or unit of dry and liquid measure; and a measured ounce of pure water at standard (i.e., mean) circumstances is the unit of weight, commonly called an ounce weight. Thus, the primary unit has a volume of $\frac{32}{57}$ that of the unit of capacity, and a weight of $3\frac{1}{5}$ ounces. The sum of its linear edges is $12''$, or one foot; and its total surface is 6 square inches $= \frac{1}{24}$ a square foot. $\frac{5}{16} = .3125$ of the primary unit is an ounce weight; and $\frac{57}{32}$ the volume thereof—i.e., of the primary—is the unit or ounce of capacity measure, etc. The aliquot parts of this primary unit are 1,536 grains, or 2,000 ultimates, the cubical and square properties of which numbers relate them to all geometrical volumes and surfaces; while, by their roots (8 and 10), the whole Anglo-Saxon system based thereon is made to partake of all the advantages of both the decimal and octenary systems of notation. Thus, $1536 = 3(512)$, and $2{,}000 = 2(1{,}000)$, or $1{,}536 = 3(8)^3$, in which 8 itself is also a perfect cube, and $2{,}000 = 2(10)^3$. From the cubical, even, and triple nature of the first may be realized all the desirable aliquots in common-fractional form; and from the even and cubical form, and the decimal root of the latter, all the necessary decimal fractions of

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[1] Should, however, a standard density of more or less than 5.7 (exact) be desirable,—as, for instance, one of $\frac{180}{10\pi} = \frac{180}{31.4159} = 5.729 \pm$ etc.—then the relations of the system to the standard cube of $50''$ on an edge will result in proportional figures. Thus, $125{,}000 \times \frac{180}{10\pi}$ will equal $716{,}187 \pm$ cubic inches of pure water at standard circumstances, and all the figures throughout the system would correspondingly vary. For obvious reasons, however, I am of the opinion that the standard density (5.7) without further fractional termination, and because of its simple relation to the cubic volume 712,500, through the cube of $50'' = 125{,}000''$, is the proper one to employ.

weight, etc. Again, $1,536 = (16)^2 6 = 2(2 \times 8)^2 3$, and $2,000 = 5(20)^2 = 5\ (10 \times 2)^2$,—factors which exhibit respectively the even, square, circular, octenary, and triple properties of the one, and the quintuple, square, circular, decimal, and even properties of the other. All of the above properties of the primary unit connect it with number, length, surface, volume, capacity, and weight, not only as elements of geometrical form, as such, but as Anglo-Saxon elements, and as elements that are universal, since they are terrestrially commensuric,—the inch (linear) being the one 500-millionth of the polar axis; and the density, 5.7, being the mean of earth density, etc.

STANDARD PHYSICAL CONDITIONS.

For international and earth-commensuric metrology, the standard physical condition for every variable of nature (such as temperature, atmospheric pressure, hygroscopic state, etc.) concerned therein, shall be the *mean of all the earth* for that variable. Expressed in degrees, upon the respective scales for each of the several variables, this mean shall be indicated by 50°; and when any particular variable—as, for instance, temperature—is being considered alone, all the others—such as atmospheric pressure, hygroscopic state, etc.—shall be maintained at their mean, 50°, each upon its proper scale. Thus, when testing and stamping the weights, measures, etc., of earth-commensuric metrology, the international observatory shall be at the mean temperature, pressure, hygroscopic state, etc., of the whole inhabited earth, or shall be at 50°, from which they will vary less, and around which they will be handled most by man, from day to day throughout the year, and here and there upon the earth. These standard or mean physical conditions are now believed to be as follows:—

Standard or Mean Temperature equals + 68° Fahrenheit = + 16° Réaumur = + 20° Centigrade, or Celsius, or, upon the international scale, = + 50°, on which same scale water freezes at 0°, and boils at 250°. Standard temperature is thus one-fifth the way from freezing to boiling point of water.[1]

[1] Very beautiful and accurate thermometers of this graduation may now be obtained of J. S. F. Huddleston, 242 Washington Street, Boston, Mass. They are made expressly for the Institute.

Standard or Mean Atmospheric Pressure is that indicated by 30 axial inches of barometric mercury (specific gravity = 13.557), other physical conditions being standard. This height, at the sea-level on the standard parallel (30° north latitude), corresponds to 30.03 present British inches of mercury (at specific gravity 13.598) 4° C., and on the international scale is marked by 50°; 48° thereon corresponding to 28″.8, and 52° to 31″.2; 0° = 0″, and corresponds to "no atmosphere." A substance as light as cork (specific gravity .24; i.e., 15 pounds per cubic foot) would stand 1,694″.58 high in the barometric tube, or upon such a scale balance the atmosphere at 2,824°.375; and one as heavy as hammered iridium (taken at specific gravity 24) would stand thereon at 28°.24375 = 16″.9458.[1]

Standard or Mean Hygroscopic State of the Atmosphere is 50°, on a scale where 0° marks "no moisture;" and 250° indicates saturation, other circumstances being standard. The hygroscope stands thus at one-fifth the way from dry to wet, and indicates a dew-point at 18°.055 + etc., on the international thermometer (Fah. 68°, dew-point 45°). This standard hygroscopic state is marked upon the hygrometer by an amount of aqueous vapor in the air equal to $\frac{1}{136}$ its volume, or equal to 4″.3 cubic inches of water per column of air one square inch at base, and equal in height to the atmosphere. Its pressure = $\frac{1}{95}$ that of the whole atmosphere.[1]

Standard or Mean Density is taken as 5.7 times that of water at other standard circumstances. This density being indicated by 50°, water will be indicated by 8°$\frac{44}{57}$, hammered iridium (specific gravity 24) by 210°$\frac{30}{57}$, hydrogen (specific gravity .0000895 ±) by °.000785087 + etc., and a substance whose density is 28.5 by 250°, etc., thereon.

SPECIAL STANDARDS.

Under the foregoing standard circumstances (i.e., the metrological chamber of observation being brought to 50° upon each of the international scales), the following are the standards in particular: —

[1] Arrangements are now being made with instrument-makers of standing for the manufacture of barometers and hygroscopes graduated upon this system, and will be placed with the Institute for sale.

Standard of length is one 10-millionth of the semi-polar axis, is called a cubit, and equals 25 earth-commensuric inches = 25 present Anglo-Saxon inches + .001 (±).[1]

Standard of volume is that of a cube whose edge is two cubits; i.e., $= (50'')^3 =$ (125,000 cubic inches) for geometrical purposes.

Standard of weight is a standard volume of mean density material; i.e., it is the weight of a cube of material whose density is 5.7 times that of water, and whose edge is 50″. This cube weighs 400,000 ounces $= 50(20)^3$ ounce weights: hence it is equal to 50 times a cube which weighs 8,000 ounces. Such a cube as the latter, if its volume were 8,000 cubic inches, would have to be of a density equal exactly to 1,781.25, water being 1,000. This is related to all Anglo-Saxon weights. For practical cases of *comparison* this standard cube of 125,000 cubic inches, mean density material, may be divided into 1,000 cubes of 5 inches on an edge. Each such cube of comparison will thus contain 125 cubic inches, mean density material, and weigh exactly 400 ounces = 25 avoirdupois pounds, 33½ troy and apothecaries' pounds, 40 standard or 10-ounce pounds, and be of handy size for comparative metrological purposes.

Standard of capacity is 71,250 earth-commensuric cubic inches, or one ton. It is the volume occupied by 40,000 ounces of pure water under standard circumstances. Numerically it is expressed, $\frac{1}{10}\,(50'')^3\ 5.7 = \frac{1}{10}\,(125{,}000)\ 5.7 = \frac{1}{10}\,(712{,}500) =$ 71,250 cubic inches pure water, etc. All dry and liquid measures of the rectified Anglo-Saxon system run up to and through this standard.

UNITS AND PRIMARIES OF SYSTEM.

Primary temperature. — Freezing-point of water = 0° on international scale (= 32° Fah., etc.).

Primary barometric point. — Vacuum = 0° on international scale (= 0″ of mercury, etc.).

[1] The standard cubit, of 25 inches, graduated in various ways to accomplish the several geometrical problems described in Appendix A, are soon to be manufactured by Darling, Brown, & Sharpe, of Providence, R.I., and will be within reach of such as desire to obtain them.

Primary hygroscopic point. — Dry air, or no moisture, $= 0°$ on international scale.

Primary density point. — That of vacuum = specific gravity $0.0 = 0°$ on international scale.

Unit density = that of water, pure, air at the international standard (50°) points, as to temperature, pressure, and hygroscopic state, and on the international scale of density corresponds to $8°\frac{44}{57}$.

The unit of length, or lineal unit, $= 1''$ (inch) = one 500-millionth polar axis = 1 Anglo-Saxon inch + .001 (±).

The unit of area, or square unit, $= (1'')^2$ = a square on one unit inch linear.

The unit of volume, or cubical unit, $= (1'')^3$ = a cube on one linear inch.

The unit of capacity measure (dry and liquid), or the capacity ounce, = 1.78125 (exact) units of volume = space occupied by one ounce weight of pure water under standard circumstances, or that occupied by 5.7 ounce weights of mean density material.

The unit of weight, or ounce weight, = weight of 1.78125 unit volumes of pure water at standard circumstances, or weight of $\frac{5}{16}$ a unit volume of the mean density material = 1,536 grains = 2,000 ultimates.

CONSTANTS OF SYSTEM.

The term standard = 50°, on every scale, = mean of all terrestrial or physical circumstances concerned in metrology; i.e., = perfect health as to temperature (68° Fah.) = 5.7 specific gravity as to density, 30″ as to barometer, $\frac{1}{5}$ as to saturation, etc.

Unit inch = one 500-millionth of polar axis of earth = 1 Anglo-Saxon (A.D. 1883) inch, + .001″ ± to rectify it.

Unit square $= (1'')^2$.

Unit volume $= (1'')^3$.

Unit cube $= (1'')^3$, mean density material.

Unit ounce weight $= 1\frac{57}{320}$ (= 1.78125, exact) cubic inches, or unit volumes of water, standard; $= \frac{5}{16}$ (= .3125) cubic inches, or unit volumes of mean density material, standard; = 480 grains = 625 ultimates.

Ounce of capacity = 1.78125 (exact) unit volumes, standard,

= space occupied by one unit ounce weight of water, standard, or by 5.7 unit ounce weights of mean density material, standard.

1 grain = $\frac{1}{1536}$ of unit cube = $\frac{125}{96}$ ultimates = .0037109375 (exact) cubic inches of pure water, standard.

1 ultimate = $\frac{1}{2000}$ of unit cube = .00285 (exact) cubic inches pure water, standard.

The foregoing being premised, it is proposed, that, in addition to the several systems of Anglo-Saxon weights and measures now in use (and which shall all be rectified thereby), the several tables heretofore designated as "standard" shall be made legal for decimal and other purposes, but that no system shall ever be made compulsory. This having been accomplished, and all the tables being united by a common inch, ultimate, grain, ounce, and gill, and by the process of rectification being made strictly accurate, scientific, and earth-commensuric, either system of subdivision may be employed, according to the demands of special arts, sciences, and trades, and the several systems shall be left to work out their destiny according to their several inherent merits.

In order to introduce the standard or *decimal* systems here proposed, and which are founded respectively upon 10 units of each of the several rectified Anglo-Saxon tables now in use, it will by no means be necessary to adopt the entire scheme of the new tables as given in this volume. All that is actually essential is, that 10 inches shall be declared and understood to mean a standard or decimal foot; 10 ounces a decimal pound; and 10 capacity ounces (= 17.8125 cubic inches) a decimal pint, etc. Other multiples of the above, such as by 10^{s}, 100^{ds}, 1,000^{ds}, etc., may, or may not, be especially mentioned. But, be they mentioned or not, they will, nevertheless, all be logical and potential parts of the system, and as such be within reach of those who may desire to employ them, and be intelligible to those who read them. The whole point of this discussion is to be summed up in the statement, that it is perfectly practicable for us to introduce the decimal system into our own weights and measures, and (if we rectify at the same time our units so as to comprehend cosmic principles) thus found a system of metrology that can never more be shaken or disturbed.

VI.

AEROSTATICS.

"The firmament showeth his handy work." — Ps. xix. 1.

The Atmosphere in its Relation to the Rectified System. — Its Weight. — Specific Gravity. — Mean Barometric Height. — Dew-Point. — Moisture. — Actual Height of Atmosphere.

It is in the atmosphere that "man lives and moves, and has his being." The business of his life is in weighing and measuring the things about him; and, to do so accurately, a thorough knowledge of the condition and changes of the atmosphere, and of their corresponding effects upon metrology, is absolutely necessary.

Viewed from the standard observatory of this new system, we will briefly enumerate a few of the more important facts and data which are concerned directly in this subject.

A cubic foot of atmospheric air at the surface of the earth (mean sea-level) and at standard physical conditions, i.e., 50° upon each of the international scales (68° Fah., 30″ mercury at 13.557 density, $\frac{1}{6}$ saturation, etc.), weighs 570.2809536 +, etc., grains.

Its specific gravity, compared with water, pure, and at standard circumstances, is .0012246970 +, etc.

A column of air at the circumstances which would exactly balance an equivalent (as to cross-section) column of the atmosphere itself would be 332,100 inches high, or 5.2414828 miles. This, in other words, would be the height of a barometric column of air if it could be maintained throughout its height at standard circumstances. It corresponds to a barometric height of water, at standard conditions, equal to 33.8925 feet, or to 406.710 inches Taking all of these barometric columns as of equivalent cross-section, and that cross-section one square inch, the pressure indicated would be 14.27052631 +, etc., avoirdupois pounds (i.e., rectified pounds at 28.5 cubic inches, pure water, standard circumstances, per such pound). Expressed in troy or apothecary pounds, this pressure is 19.02736841 +, etc., pounds; and in standard or 10-ounce pounds, 22.832842096 +, etc., pounds. Expressed in grains, this pressure is 109,600; and in ounces, the most convenient and numerically satisfactory form for use, it is 228⅓ ounces, almost exactly.

At 50° standard thermometer, hygroscope, and barometer, the standard dew-point is at 18°.05555 +, etc., temperature. This indicates that the total barometric force of the moisture (aqueous vapor, or steam) of the air is about $\frac{1}{95}$, or, more accurately, $\frac{1000000}{94902924}$, of the whole atmospheric pressure. This moisture exists in the form of atmospheric steam (specific gravity .00086483342), and occupies about 89,792 times the volume of water which would produce it. Condensed to water, the total amount of aqueous vapor in a standard column the height of the atmosphere, and a square inch in cross-section, amounts to 3.698546 cubic inches. There are .003000 grains thereof per cubic inch of mean atmospheric air, or about $\frac{1}{110}$ (more accurately, $\frac{1000}{110079}$) the weight thereof. In general terms, 110 times the actual weight of the moisture present in standard air is equal to 95 times the pressure which it exerts in the form of steam. Thus, weight when condensed : pressure in vapor :: $\frac{10000}{1100079} : \frac{1000000}{94902924}$.

The maximum (250° hygroscope) capacity of air at otherwise standard circumstances for moisture is .015 grains per cubic foot, 1,875 grains per standard cube, etc. The mean hygroscopic state is .003 per cubic inch, 5.184 per cubic foot, and 375 per standard cube.

A cubic inch of atmospheric air at standard circumstances weighs .3300237 grains, .003 grains of which we have already seen to be of aqueous vapor, leaving .3270237 grains to be divided among the other constituents thereof. Putting the weight of air, as such, into a simpler and more beautiful form, a standard cube thereof weighs 5½ rectified troy pounds (accurately this is 5.500395 ± pounds troy).

The relation between air at mean, and water thereat, is expressed by their specific gravities, .001224697 : 1.0000, or 816.520331 +. That is, air is 816.52, etc., times lighter than water, the material of unit (1) density, and 46,540.64 times lighter than the standard density material (specific gravity 5.7 times water).

13.4670462, etc., cubic feet of air weigh a pound avoirdupois (rectified).

The rate of expansion of air from 50° to 250° standard thermometer is uniform, and equal to .0015034 of its bulk at 50° for every standard degree of heat. As the temperature descends below 50° to the 0° point, air decreases in volume uniformly, and at the same rate, from its standard bulk at 50°. Above 250° and to 1,000° it increases in bulk at the uniform rate of .00144 of its volume per every degree of heat.

Air at 50° temperature, when completely saturated with moisture, will hold .015 grains of atmospheric steam per cubic inch. This corresponds to 18.492730 cubic inches of water per mean atmospheric column (i.e., 1 inch in cross-section, and 332,100 feet high).

The actual height of the atmosphere has been variously estimated. It has been generally estimated at 53.3 miles. This, however, is now regarded as far too small. Of course, the atmosphere has a limit, and that, relatively to the radius of the earth, a very narrow one. Its mean height — that is, the whole atmosphere reduced to a standard condition such as it is at sea-level when all the international scales are at 50° — is, as we have seen, 5.2413 + miles, which is about $\frac{1}{750}$ the mean radius of the earth. Now, Monsieur Liais found, by experiments on the twilight arc at Rio Janeiro, that the height of the atmosphere was probably in the neighborhood of 212 miles. The indications are, that about $\frac{1}{18}$ the radius, or 220 ± miles, certainly includes its extreme height.

When the altitude of the atmosphere is taken in arithmetical progression, its rarity is found to be in geometrical progression. Thus, 5.24132 +, etc., miles being the mean height at standard circumstances for sea-level conditions, at twice that height, or 10.48262 miles, the air would be 4 times as rare, at 20.96 miles 16 times as rare, etc. This will best be seen from the following table : —

TABLE OF ATMOSPHERIC HEIGHT AND RARITY.

If	5.24132 miles high, rarity			=	1 (i.e., unity).				
"	10.48264	"	"	"	=	$\frac{1}{4}$	times as rare.		
"	20.96528	"	"	"	=	$\frac{1}{16}$	"	"	"
"	31.44792	"	"	"	=	$\frac{1}{64}$	"	"	"
"	41.93056	"	"	"	=	$\frac{1}{256}$	"	"	"
"	52.41320	"	"	"	=	$\frac{1}{1024}$	"	"	"
"	62.89584	"	"	"	=	$\frac{1}{4096}$	"	"	"
"	73.37848	"	"	"	=	$\frac{1}{16384}$	"	"	"
"	83.86112	"	"	"	=	$\frac{1}{65536}$	"	"	"
"	94.34376	"	"	"	=	$\frac{1}{262144}$	"	"	"
"	104.82648	"	"	"	=	$\frac{1}{1048576}$	"	"	"

That is, at 20 times the mean height, or 104.82648 miles, the rarity of the atmosphere would be expressed by a fraction whose denominator was $(2)^{20} = 1,048,576$, a number which is practically equal to the height in miles 104.8264 multiplied by 10,000. The specific gravity of air at this degree of rarity would, under other standard circumstances, be .000000001 16795+, etc. Furthermore, at double this height, or 209.65296 miles, the rarity would be $\frac{1}{(2)^{40}} = 1 \div 1,099,577,163,776$; and at 220 miles, which we have just considered to be practically the absolute limit of the atmosphere, the rarity would be just short of $\frac{1}{(2)^{42}}$ of air in the standard observatory, and at standard circumstances. Numerically this may be put as follows: at 42 times the height (5.241, etc., miles) of a standard atmosphere, the actual rarity of air (free to occupy that space by the atmospheric law above noted) would be expressed by $\frac{1}{(2)^{42}}$. 42 times 5.241, etc., miles = 220.1356 miles, and $(2)^{42} = (4)^{21} = 4,388,308,655,104$, say practically, for the rarity 4,400,000,000,000, or equal to a trifle less than twice the height in miles multiplied by $(100)^5$, i.e., extreme rarity $= \frac{1}{2(220)(100)^5}$.

Now, the atmosphere is a sea of air, whose fluctuations at a surface of such extreme rarity must produce waves of enormous size, and waves that (from the nature of gases, earth motion, and the earth-surface atmospheric commotion) can never be at rest. Taking these figures, therefore, as representing, as closely as possible, the actual mean height of the fluctuating atmospheric surface of the earth, the specific gravity of air at this surface will be represented by the standard specific gravity of air (.001224697) divided by $2(220)100^5 = .00000000000002$-

78340; i.e., one quadrillion times as light as a density 2.78, or about one quadrillion times lighter than that of alabaster, chrysolite, aluminium, and chalk.

At this limit the atmosphere may certainly be regarded as being in equilibrium with the ethereal medium of space, than which nothing rarer probably exists in nature.

VII.

CONCLUSION.

"Felix qui potuit cognoscere causas."

Metrology the Universal Science.—There is no Exception to the Law of Number. —Man's Love of Exact Numbers.—No Such Thing as "Mere Coincidence."—How Nature replies to us in Terms of the New Metrology.—The Standard Cube.—Solomon's Wisdom.—But these Facts are Older than Solomon or Moses.—The Great Pyramid.—The Standard, or King's Chamber.—The Sanctum Sanctorum of all Science.

It is the highest pleasure of the finite mind, to be allowed to discover the ways of the Infinite in the ordering of the universe:—

"Felix qui potuit cognoscere causas."

When man has thus discovered facts which correlate themselves to the constitution of all nature, he calls them science; and it is only when they do thus satisfy the demands of eternal fitness, that they may be classed with "science properly so called."

Metrology is the universal science; because He who created Wisdom, and saw her, and poured her upon all His works, hath "ordered all things in measure, and number, and weight." [1]

Thus, the system upon which creation rests is metrology itself. Nor is the attempt to discover this science a hopeless effort. Man has already made vast strides in the right direction; and the Anglo-Saxon system of to-day, from whencesoever it was originally derived, has been shown to be but slightly from the truth. In botany, man has learned that all vegetable-life, from the cell to the leaf, is but a numerical arrangement of cells; and from the leaf to the tree is likewise

[1] Wisdom xi. 30; Eccles. i. 9.

simply one of leaves. In crystallization, it is a law of numbers, and a rigid one, that governs. On it (number) the whole of chemistry is founded; and so, too, with every branch of science. These laws — the special ones of each branch of science — are all simple: there are no complex general laws. The more general the law, the simpler its numerical expression. Just as force is correlated to all other kinds of force, so, too, all forms of matter are undoubtedly likewise correlated; and whether it be in planetary forms, or those of art and trade, they are ruled by simple numbers.

Throughout his practical life, man displays a universal tendency to employ "round numbers," as his factors of use, in dealing with physical things. This tendency is rooted, not merely in a desire to shorten his calculations, but still deeper, — in an innate conviction that the beauty and symmetry existent in the constitution of all nature consists in its simplicity, and particularly in the simplicity of its numerical expression. Hitherto it has been the inaccuracy of our determinations, in the field of universal science, that has introduced so many irregular and awkward quantities into our tables of physical data; and, until this inaccuracy has been overcome, we cannot feel that any branch of modern science is upon a firm and eternal basis.

But progress is slow. The prejudice of long use, and the natural conservatism of philosophers, conspire together against every change, as such, no matter how great the promised benefit therefrom may be. Nevertheless, the stronger tendency to simplicity, and the deeply rooted conviction that it exists, — if it can but be discovered, — govern practical life, and, when the true secret of numerical arrangement is revealed, will overcome all opposition, and round, perfect, and beautify our scientific data by its own exalted standard.

Among some of the most familiar examples of man's deference to round numbers, we shall mention but few. Thus, under the present system of Anglo-Saxon metrology, though 14.72 + pounds per square inch is the tabulated pressure of the normal atmosphere, it is habitually taken at 15 pounds. The present Anglo-Saxon ounce is not earth-commensurable; and hence neither it, nor its multiples, will necessarily greatest commonly divide, nor least commonly multiply, the forces of nature. Man, therefore, has to be inaccurate in his practice, for the sake of convenience, and in some of his most important mechanical works, as those concerned with steam, call 14.72 +, etc., arbitrary pounds, 15. Take, moreover, the weight of a cubic foot of water. This is habitually regarded as $62\frac{1}{2}$ pounds; though it is, so accurately as known, by the old system of metrology, 62.379, etc., pounds per cubic foot. Then, there is the number $\pi = 3.141592$, etc., which we habitually take at 3.1416, and so on throughout all science.

Now, it cannot be that Nature herself is thus inaccurate, nor that she is similarly irregular in her ratios, factors, and moduli. And it must be, moreover, that, in a realm where "all things are weighed and measured in number," there is a universal law of number harmoniously running through the things so weighed and numbered, and that this law may, in due time, perhaps, be discovered by man, who is himself its highest exponent.

Now and then some faint glimmering of this numeric symmetry appears, but it is merely regarded as a coincidence. Instead of being studied with a view to following up the clew, and reaching on to the universal system itself, these coincidences are hardly ever tabulated, or even noticed, in what are considered to be strictly scientific works; although, whenever known, they are universally employed by the practical man. Astronomy is full of them, but only because they are more apparent in the wider sweep of its grand quantities and cycles. As an actual fact, however, astronomy is no more occupied by coincidence, as such, than any other branch of science. They all teem with it; and, to their deeper students, it crops out at every turn. Coincidence is a law — indeed, it is the grand numeric law — of nature. It is based upon certain general underlying principles of number, weight, and measure, universally pervading things as they are; and there is no such thing as mere coincidence, or as accident, in its commonly opprobrious sense.

Let us now briefly consider Nature from the stand-point of the new metrology, and ask her some of the same questions which she so uncertainly answers in terms of any of the arbitrary systems hitherto in vogue.

She gives us, as the pressure of the atmosphere, $228\frac{1}{3}$ ounces, — an accurate as well as satisfactory and round number. This can be put into any kind of pounds, or used in grains at 109,600, or in cubic inches of water at 406.710, or of mercury at 30″, or in air at standard condition, as 332,100 cubic inches, etc. She gives us, for the weight of a cubic foot of water at standard circumstance, $970\frac{6}{57}$ ounces exactly, or $97\frac{1}{57}$ standard (10-ounce) pounds. It is even being ably urged, by those who are skilled in the logic and philosophy of geometry, that she gives us for the true circummetric ratio the common fraction $\frac{20612}{6561}$, and pronounces against the old value of π. She gives us, as the mean height of the atmosphere in miles, $\sqrt[3]{144} = 5.2414828$, etc. (long measure rectified). She gives us, as the number of cubic inches in a vertical square-inch column of this mean atmosphere, $63{,}360 \times \sqrt[3]{144}$; in which 63,360 is the number of standard inches in a rectified mile (long measure). She gives us, as the weight of a standard cube $(50'')^3$ of mean atmospheric air, $5\frac{1}{2}$ troy pounds, rectified, accurately (5.500395+, etc.).

But let us examine this system still more closely, and, to do so, consider the weight of $(50'')^3$, or 125,000 cubic inches of standard atmospheric air. Since its specific gravity is .00122469, etc., or since one cubic inch weighs .3300237 ± grains, it follows that $125{,}000 = (50'')^3$ cubic inches will weigh 41,252.9625 such grains. Now, at the first glance, this is an ordinary series of figures; but, upon examination, it proves to be a most extraordinary one. Upon the Babylonish system of circular division, it is one-fifth of the radius expressed in terms of seconds of arc. Thus, in 180° there are 648,000 seconds, $648{,}000 \div \pi = 206{,}264.8125$, etc.; and $\frac{1}{5}$ of this latter number $= 41{,}252.9625$, etc. If, upon the other hand, we take the subdivision of the circle into 240°, a division for which the Great Pyramid pronounces in no uncertain terms as the only correct and rational one, and as one which is closely in keeping with the requirements of general metrology, the number 41,252.9625, etc., is even more expressive, and may be written thus: —

$$41{,}252.9625, \text{etc.}, = \frac{240^\circ}{5}\,\frac{3^3}{\pi}\,10^2 = \frac{(2)^4(3)^4}{\pi}\,10^2 = \frac{2^6\ 3^4\ 5^2}{\pi} = \frac{10^2\ 6^4}{\pi}.$$

Now, all this is called coincidence in common parlance; and so it might have been had our grain been an arbitrary thing, such as the present unrectified Anglo-Saxon grain, or the metric gram. But the grain we are using is not arbitrary. It is directly related to the grandest linear dimension of the earth, to its mean density, to its cubical units of capacity and weight, and to the specific gravities of all substances as correlated in water as unity, mean density material as standard, lead as our synonyme for weight, iron as our practical weight material, air as the element in which we live and move, and to the properties of geometric form, and the elementary principles of numerical expression. Of course the cube, on 50″ as an edge, is but one out of an infinity of cubes that might have been selected; and air is but one of a myriad of substances. Nevertheless, it is a cube already chosen from other and general earth-commensuric and numerical principles, as our standard one, and certainly, in the light of such remarkable relations as those with which we have found it pregnant, well worthy of its place.

From the converse of the foregoing discovery, it follows, that, so nearly as we can grasp it (and, from the standpoint which we occupy, we believe it to be so absolutely), the weight in grains of standard air per cubic inch is expressed by the term $\frac{8}{6}\left(\frac{648}{1{,}000\pi}\right) = \frac{1}{2\pi}\left(\frac{6}{5}\right)^4$ grains.

This is the discovery of a fact in nature. It is true, of course, only for that particular value of the grain which we have found reason to adopt; but this is a value which rests upon other and entirely independent earth-commensuric considerations.

Now, the fact that an atmospheric cube of 1 terrestrial inch on an edge, contains, at standard, or mean earth conditions, a number of earth-commensuric grains expressed by the remarkably circular and circummetric function, $\frac{8}{6}\,\frac{648}{1{,}000\pi} = \frac{120^\circ}{\pi}\,\frac{(3)^3}{(5)^5}$, is of the utmost importance to metrologists. It is a common term, as it were, to a vast number of independent series of physical data, — a point in which all of these various sequences coincide and harmonize, or through which they pass in progress from 0 to infinity.

To man, so devoted a lover of the beautiful, a fact like this is very impressive; and it seems to have governed the whole metrological system of the ancient Hebrews, — a system, be it remembered, to which a most lofty origin has been universally accorded. The fact may, indeed, have been connected with the very origin of the grain itself. At any rate, it was too important a common tangential point of all of Nature's physical data, and of her laws of numerical expression, to be overlooked; and so it was given the very highest prominence at Jerusalem.

We have already seen sufficient reason for regarding the most sacred precinct of Solomon's temple as a metrological observatory. Whatever were its special religious uses, it was also a national chamber of weights and measures, and one whose equable atmosphere was weighed and measured and numbered in terms of this very expression, $\frac{120^\circ}{\pi}\,\frac{(3)^3}{(5)^5}$.

The Holy of holies was a perfect cube, each of whose edges was 20 sacred cubits, or 500 inches long. Hence this sacred cube — the ancient God-designed and Israelitish metrological chamber — contained a volume of 125,000,000 cubic inches of atmosphere, or by weight, at standard circumstances, $(500)^3 \frac{120^\circ}{\pi}\,\frac{3^3}{5^5} = 500^3\, \frac{8}{6}\left(\frac{648}{1{,}000\pi}\right) = 41{,}252{,}962$, etc., grains.

This, on the Babylonian division of the circle, is numerically equal to 200 times the length of radius in seconds of arc, or to 1,000 times as much as we have found in a standard cube. Again, this expression for the weight of the atmosphere of the Holy of holies in grains, is $\frac{1}{5}$ the aggregate length in seconds of arc, of the radii upon 10,000 circles; or, even more concisely, it is equal to the number of seconds in 100 circumferences ($36{,}000^\circ = 129{,}600{,}000''$) divided by π.

Now, it seems to be the conviction of a very large class of American pyramid students, that the number $1{,}296{,}000 = 360 \times 60 \times 60$, and its various derivatives, 648,000, 324,000, etc., necessarily refer to the division of the circle now in universal use among mathematicians, and that it can have no reference to any other division. This is entirely an error, and has arisen from the fact that they also persist in looking upon the Great Pyramid as a monument erected to perpetuate *direct* measures, rather than to preserve cosmic *ratios* by means of certain standard lengths. This error has been repeatedly pointed out in "The International Standard," once by the author (vol. i., No. 3, July, 1883, p. 208, lines 30, 31), then by Jacob M. Clark, C. E. (vol. i., No. 5, November, 1883, p. 332, lines 3–6), and finally by Mr. J. Ralston Skinner (vol. i., No. 6, p. 519, lines 46–48). Indeed, as Mr. Skinner implies, the idea seems to have been grasped by Professor Smyth long ago, though certainly not clearly enunciated by him. Misled by the intentional concealment of these ratios here and there in *inches* (and many of the most beautiful of them are undoubtedly so expressed — but in *earth-commensuric* or *pyramid inches*), and entirely ignoring their frequent repetitions in the sacred cubit of 25 such inches, and believing these inches to be our present Anglo-Saxon unit, this school has built its entire theory upon the division of the circle into 360°. But there are insuperable objections to this division of the circle. It can form no part of the geometric scheme of the Great Pyramid, if that edifice is indeed an embodiment of accurate metrologic principles.

Once establish the premise that the pyramid is an exponent of "just weights and *perfect* measures," and that its erection was supervised by an architect so inspired as to have been incapable of erring as to the proper number of aliquots in a circle, and it immediately follows, that the $360° \times 60' \times 60''$ division could not have been the basis, since it is irrational. No means is known to geometry of obtaining a 360th of a circle, or of an angle; it involves trisection: and we shall "square the circle" so soon as we accomplish it. We can only obtain such a division by the process of infinite approximation in practice, and in theory have long ago given it up. One of the simplest modes of establishing the theoretic impossibility of obtaining the $\frac{1}{360}$ of a circle is as follows: $5° = \frac{1}{72}$, $6° = \frac{1}{60}$, $6° - 5° = 1° = \frac{1}{60} - \frac{1}{72} = \frac{12}{4320} = \frac{1}{360}$.

Now, if we can construct the $\frac{1}{72}$ and the $\frac{1}{60}$ of a circle, we can by comparison obtain the $\frac{1}{360}$. But while by two bisections from the $\frac{1}{15}$ (which is itself obtained from comparison of the $\frac{1}{6}$ and $\frac{1}{10}$, both rational) we can obtain the $\frac{1}{60}$, we cannot similarly obtain the $\frac{1}{72}$. This fractional part of the circle depends upon the $\frac{1}{9}$, and is the $\frac{1}{8}$ thereof, or might be obtained from the

$\frac{1}{9}$ by three bisections if we could only obtain the $\frac{1}{9}$. But the $\frac{1}{9}$ is *irrational:* hence the $\frac{1}{360}$ is equally so. This is usually lost sight of because the number 360°, being divisible by 9, i.e., giving a quotient 40°, the habitual rule of the text-books is to lay off 40° (see Trautwine and others), and its chord is that of the ninth. Who cannot see, however, that this is merely begging the question? for to use a circle only approximately subdivided into 360ths by the instrument-maker, as a means of geometrically (!) constructing an accurate 9th, is arguing literally "in a circle."

Geometry having therefore no *positive* means of so subdividing a circle, we must abandon the method in the interests of true metrology, as in time we shall have to abandon all other methods which we have derived from Babylon.

The methods of Babylon are so pregnant with the numeral 6, that, in all her "times and seasons," her measures and her weights, her name and number, have from time immemorial been branded far and near as 666. It is the prevalence of this numeral 6, trailing, *like the serpent it represents,* over all of our inheritance, that makes our modern metrology a science now so thoroughly "at sixes and sevens" with itself. $6 + 7 = 13$ has always been regarded as a number of rebellion, and not until we have thoroughly purged our science of the dominant factor 6 can we hope to make any progress towards the numbers of perfection. There certainly can be nothing in common between the Tower and system of Babel and the Pyramid and earth-commensuric system of Gizeh. These monuments were the very exponents of two diametrically opposite classes of ideas. The one was marked for endurance, the other was instability itself. Harmony pervaded the one: confusion presided at the other. Promethean success crowned the capstone of the one: the other was the Epimethean experiment of a man whose very name has since become the synonyme for rebel. The one was the wise man's building, *literally* "founded on a rock;" the other that of Folly herself, built upon the sand, and out of slime and clay.

So diverse, in fact, was the genius that governed the metrology of Babylon from that of the Hyksos builders of the pyramid, — men, let it be remembered, who "came out of

Chaldæa" in every sense of the word,[1]—that, just so sure as it was undoubtedly Babylonian to subdivide the circle into 360°, so we may be certain that it forms no part of the true system of metrology as monumentalized at Gizeh.

But the number 1,296 and its derivatives are by no means dependent upon the 360° circle and its sexagesimal subdivision: they come as directly from 12, 24, 240, etc. Thus, 1,296 is the number of inches in 108 feet, or it is the number of hours in 54 days. Let us examine its reference, by way of further argument, to the *rational* division of the circle into 240°. For those who are not familiar with the overwhelming arguments in favor of this beautiful subdivision, Appendix B has been written by Jacob M. Clark, civil engineer of the New-York Central Railroad, to whose erudition we are in modern times solely indebted for the discovery.

Now, at the first glance it is manifest that any ratio, depending upon the number 360, is equally related to the number 240, since both are multiples of the common factor 120. However, to proceed with the discussion, let us examine the table proposed by, and founded upon the discoveries of, Mr. Clark.

RATIONAL CIRCULAR MEASURE.

Circle.	Sectors.	Deg.	Min.	Sec.	Thirds.	Fourths.	Fifths.	
⊕	▷	°	′	″	‴	iv	v, etc.	
1 =	24 =	240 =	2,400 =	24,000 =	240,000 =	2,400,000 =	24,000,000 =	etc.
	1 =	10 =	100 =	1,000 =	10,000 =	100,000 =	1,000,000 =	etc.
		1 =	10 =	100 =	1,000 =	10,000 =	100,000 =	etc.
			1 =	10 =	100 =	1,000 =	10,000 =	etc.
				1 =	10 =	100 =	1,000 =	etc.
					1 =	10 =	100 =	etc.
						1 =	10 =	etc.
							1 =	etc.

Now, by examining the number under discussion, 129,600,000, in the light of the foregoing table, it will be seen to be far more expressive than when viewed under the light of the sexagesimal system.

[1] "Ur," the Chaldæan name for *Great*, was a designation of the Tower of Babel (see Dr. Redfield in "International Standard" for March, 1884): so that most significantly we are informed that Abraham, the father of the faithful, came out from "Ur of the Chaldees;" i.e., from out the very shadow of their falsely conceived and uncosmically proportioned monument and system.

In the rational circummetric division this number is a concentrated symbol of the pyramidal system, whose potency, as we have seen, consists in its use of the number 5.

$$129{,}600{,}000 = \text{the number of fifths (v) in 5 and } \tfrac{2}{5} \text{ circles.}$$

In another form, which intensifies the significance while it simplifies the expression, we have

$$129{,}600{,}000 = \text{the number of fifths (v) in } \tfrac{27}{5} \text{ circles } (= \tfrac{2}{5} \text{ more than } \tfrac{25}{5} \text{ circles}).$$

That is, it is the number of fifths in five complete circles about the pentagon, plus the arc subtended by the side of that all-potent emblem, the *pentalpha*. The rhythm of this relation can finally be made even more apparent, as follows: $129{,}600{,}000^{v}$ is the number contained in the continuous arc which circumscribes the five sides of the pentagon five times, and one-fifth of the pentalpha once.

But there are other references centring in and about this number which make it still more significant in its connection with the atmospheric value of the Holy of holies.

While geometrically an irrational divisor of the circle, the number 360 has, from the days of Nimrod, been employed as a familiar divisor of time, or the year. This division is peculiarly a Babylonish one; and its employment by Daniel, who wrote *in* Babylon, concerning the duration of "the times of the Gentiles," under the type of Babylon as the "head of gold," is very pointed.

The Hebrew meaning of this great prophet's name, Dan-i-El, is "Judge of God." The Hebrew significance of ancient Babylon, or Bab-El, is "Gate of God." It was in the gates that judgments were rendered in Oriental nations. The Book of Daniel thus acquires a new significance as the judgments of God, enunciated by his representative judge, and issued from the very gate of judgment. Thus Daniel, wise beyond mortal standards, and called to pass such momentous judgments upon the Gentile world, was pleased to typify its *chronological* duration under familiar Babylonish terms. If we mistake not, there is also a pointed irony in this employment. His system is thus at perfect unity with itself. He uses the king as a type.

and great Babylon as an archetype, and with reference to each, as well as to the grander empire typified, throughout most fittingly employs the sexagesimal notation as its standard of duration.

Nebuchadnezzar, the antetype, is insane a week of years, or 7 years of 360 days each. So Babylon, the archetype put for the whole Gentile world, was, according to Daniel, to be blind in its own devices 7 "times." A "time" being a year of such years, gives us a period of 360 years; and the 7 times to pass over the Gentiles is $7 \times 360 = 2{,}520$ years. This latter number is doubly significant; for not only does it have the chronological import above noted, but it is one of the most marked numbers in the entire decimal system, being the least common multiple of all its digits, and likewise astronomically a synchronologic cycle of the Earth, Moon, and Sun.

Furthermore, the number 1,260 years (Daniel's 42 months of 30 years each), the half of this prophetic week, is less than 1,296, the root of the number we are studying, by $36 = 6 \times 6$, a most expressive Babylonish number; while 1,290 years (another of Daniel's most important numbers) is less than it by the root itself, 6.

In this connection let us refer to the number 1,296, which, as we noted above, was the number of hours in 54 common days of 24 hours each. Now, on calculating what part of the Babylonish year of 360 common days this period of 54 days may be, we are surprised to find that it comes out as the intensely Babylonish repetend, 6.6666 + etc., forever.

Again, as a "time" was 360 years, or the square of 360 days (i.e., 129,600 days), and as a week of times was $7 \times 360 = 2{,}520$ years, so a month of times was $360 \times 30 = 10{,}800$ years; and a "prophetic year," that is, one of 12 such months, was $10{,}800 \times 12 = 129{,}600$ years, or the square of 360 years. This number may be arrived at directly by noting, that, as a day upon Daniel's stupendous week of Gentile times was 360 years, so, 360 such days would be a prophetic year, or 129,600 common years. What, then, may we ask, is the implied duration of the millennium upon such a scale? To this we may reply, that as a prophetic year is thus 360 days, each of 360 years' duration, and as we are assured that with the Almighty "1,000 years are as

1 day, and 1 day as 1,000 years" (note the duplication of 1,000), so a millennium may be taken as at least implying 360 days $\times$ 1,000, each of 360 years $\times$ 1,000, or $\overline{360}^2 \times \overline{1{,}000}^2$ common years, or as equal to 129,600,000,000 years.

The following table illustrates these deductions:

Thus, a prophetic day	$= 360$ days.
a time $= 360$ years,	$= \overline{360}^2$ days.
a prophetic year	$= \overline{360}^2$ years.
	$= \overline{360}^4$ days.
a millennium	$= \overline{360}^2 \times \overline{1{,}000}^2$.

In all of these numbers,—of such grandly chronologic import,—the number 1,296 is intensely present; but it is rather through the relation noted above in the repetend 6.6666 +, etc., $\times$ 54 at 24 each, be the periods how large soever, than from any significance of the 360° division of the circle as geometrically correct and rational, that we must view this marked number and its significant use at Gizeh, Babylon, and Jerusalem, the three great metrological centres of former days.

It has nothing upon earth to do with the *irrational* division of the circle into 360° at 60′ at 60″, etc., but is beautifully in harmony with the whole pyramid and temple systems of metrology and symbology when viewed through the correctly geometrical division into 240° at 10′ at 10″, etc.

So, too, it utters significant harmonies in the *chronological* scheme of 360 days of 24 hours each.

Viewed in the light of such facts, the formula giving the weight of the atmosphere in Solomon's Holy of holies is indeed a wonderful emblem of the "breath of lives" which the Almighty One breathed into the soul of the temple of clay, which, at the beginning, he created in his own image.

In this connection, and as directly related to the atmospheric value of this sacred chamber, I wish to call attention to the sequences of numbers 648,000, 1,296,000, 20,626,481 +, etc.; 41,252,962 +, etc.; 3,141,592 +, etc.; 125,000,000, etc. They are all intimately and equally related to the cubes of 500″, of 50″, and of 5″, the decimal point alone moving according to the cube considered.

Now, each of these cubes, whether selected by the ancients on account of their notable numeric properties and specific-gravity relations, or not, certainly received the most marked deference at the hands of their metrologists; and, from what we have now discovered concerning them, they are still worthy of all the regard and prominence we can ourselves bestow upon them.

These are facts, and not fancies, and are *valuable* facts, both for the scientific and the practical man.

But their recognition seems to have been far older than the eras of Moses or Solomon. We have already seen that the metrology of a golden age earlier still — that of the Great Pyramid — was centred in and around the cube of 50″ of standard material at mean earth condition as to the physical circumstances of the atmosphere. Now, a mere glance into the king's chamber — the sanctum sanctorum of this edifice — shows us that it was also consummately constructed to perpetuate these same ratios. Its very relation to the cube of 50″ would have been sufficient to establish its tangency to the circular and circummetric ratios we found in the Holy of holies, were they not already there even far more boldly than they were at Solomon's Temple.

They occur in this remarkable room in an entirely different way, not at all concealed, but built into its sides and walls and volumes in a way too wonderful, in the light of all else we have discovered there, not to be a clear demonstration of intention and design completely realized. Thus, —

Its breadth $= \frac{648}{\pi} = 206.264$, etc., inches.

Its length $= \frac{1,296}{\pi} = 412.529$, etc., inches.

Its side-wall height $= \frac{648\,\pi - 1,296}{\pi} = 235.470$, etc., inches.

Its interior height of chamber $= 230.328$, etc., inches.

Height to which the granite floor rises up into the side-walls $= 5.1415$, etc., inches.

Total height of lower course of wall-stories $= 47.00890939+$.

Clear height of lower course of wall above floor $= 41.86730$.

Average height of the 4 upper wall-courses $= 47.1153$.

It is in the first place noticeable, that not only are these the identical ratios, but that the total perimeter, (412.52, etc., + 235.47, etc.) × 2, of one of the side-walls of this chamber, equals 1,296 exactly, or one 1-thousandth of a circumference of 360°, where each second is one linear inch. In the rational 240° circle, 1,296 is exactly $5\frac{2}{5}$ circles where each *degree* is an inch.

From calculations based upon these dimensions, we further learn that the volume contained by the extreme dimensions of the chamber is 20,036,252.7789, etc., cubic inches. The cubic space occupied by the granite filling which constitutes the floor of the chamber, is 437,500 cubic inches. The cubic contents of the lower portion of the chamber "marked off" above the floor by the first horizontal course of wall-stones = 71,250 × 50 = 3,562,500 cubic inches. Hence the part "filled in" below the floor, added to that "marked off" above the floor, = 437,500 + 3,562,500 = 4,000,000 cubic inches exact.

Now, in the light of the earth-commensuric, or rectified, Anglo-Saxon system of metrology, which we have been examining, this number is of marked import.

Numerically, $4{,}000{,}000'' = 4(1{,}000{,}000'') = 4(100'')^3$. $(100'')^3 = 8$ cubes of 50″: hence $4(100'')^3 = 32(50'')^3 = 4(2)^3\ (50'')^3$. Had it not been desirable, therefore, to build the chamber simply upon the circular and circummetric function $\frac{120}{\pi}$, its dimensions could have been 400 long by 200 wide, and its lower tier have been 50″ high, the first 5.46875 inches being filled in, and leaving $200 \times 400 \times 44.53125 = 3{,}562{,}500$ cubic inches $= 50 \times 71{,}250''$ marked off above the floor.

However, as the chamber now stands, it realizes both of these ends; and the number, 4,000,000 cubic inches, thus with so much the more difficulty so accurately set apart, demands a moment's consideration at our hands.

With reference to water as the material of unit density, this volume of 4,000,000 cubic inches has, as we have already seen, been particularly subdivided into two parts; to wit, 3,562,500 = 71,250 cubic inches × 50, and 437,500 = 71,250 cubic inches × $6\frac{8}{57}$. The latter volume seems to be thus set aside as though specially to enable the grandly even quantity of 50 coffers of water to be indicated. In the inter-relations, however, of the several volumes thus marked off, there is a remarkable prominence given to the number 57. For instance, 4,000,000 cubic inches $= 56\frac{8}{57} = \frac{3200}{57}$ cofferfuls; 437,500 cubic inches $= 6\frac{8}{57} = \frac{350}{57}$ cofferfuls, and 3,562,500 = 50 cofferfuls. Now, this latter volume is $8\frac{1}{7}$ times $= \frac{57}{7}$ times the volume filled with granite; or, in other words, the latter volume is $\frac{7}{57}$ of the 50-coffer space marked off above the floor. It naturally now occurs to us to inquire what this volume, the $\frac{1}{57}$ of a coffer, may be. $71{,}250 \div 57 = 1{,}250 = 10(5)^3$ cubic inches. It is, therefore, in volume and amount equal to 10 times the smaller of the several cubes, whose relations we have found to be so important. Again, the whole volume, 4,000,000 cubic inches, is seen upon examination to be just $\frac{7 \times 7}{57} = \frac{49}{57}$ of a coffer short of being exactly 57 coffers in capacity, — a quantity of water which would be balanced by the very notable quantity, 10 coffers of mean or standard (5.7) density material. We have already seen that 10 coffers of water are equal to a standard cube, or to $(50'')^3$ of mean density material. These, for the 10 coffers of mean density material here indicated, are equal to 5.7 times as much, or to $5.7\,(50'')^3$ of that material whose density is standard, or 5.7 times that of water.

References more pointed to the coffer, the cube of 50″, the standard density, 5.7, etc., with which this chamber thus almost cries aloud, can hardly be conceived. But let us examine the marked-off portions of the chamber with reference to some of the other important metrological substances of practical life.

If we consider that the entire 4,000,000 cubic inches marked off is a volume of the mean density material, it will equal, in terms of the rectified Anglo-Saxon system, the following quantities: —

4,000,000 cu. in. mean density material	=	8,000,000,000 ultimates = $(2{,}000)^3$.
	=	6,144,000,000 grains = $4\,(1{,}536)\,(100)^3$.
	=	12,800,000 ounces.
	=	1,280,000 pounds standard.
	=	1,024,000 pounds troy and apothecary.
	=	800,000 pounds avoirdupois.

Considering the volume to be one of lead, the values throughout will be just double the above.

Regarding the whole 4,000,000 cubic inches as filled with iron, the practical weight material of every-day life, the numerical relations to the rectified system become very noticeable. Thus,

$$
\begin{aligned}
\text{4,000,000 cubic inches iron} &= \text{1,000,000 pounds avoirdupois} = (100)^3 \text{ pounds.}\\
&= 1{,}333{,}333\tfrac{1}{3} \text{ pounds troy and apothecary,}\\
&= \text{1,600,000 pounds standard.}\\
&= \text{16,000,000 ounces.}\\
&= \text{7,680,000,000 grains} \left\{ \begin{array}{l} = 7{,}680\ (100)^3 \text{ grains.}\\ = 5\ (1{,}536)\ (100)^3 \text{ grains.} \end{array} \right.\\
&= \text{10,000,000,000 ults.} \left\{ \begin{array}{l} = 10{,}000 \times (100)^3 \text{ ultimates.}\\ = 5 \times 2{,}000 \times (100)^3 \text{ ultimates.} \end{array} \right.
\end{aligned}
$$

With reference to the atmosphere, at standard physical condition, the consideration of this marked-off portion is likewise fraught with interest. A cubic inch of standard atmospheric air, we have already seen, weighs $\frac{120}{\pi}\frac{3^3}{5^5}$ grains: hence 4,000,000 cubic inches will weigh $\frac{10(640)648}{\pi}$ grains, in terms of which expression all the derivative quantities may be written. Thus, in *ounces*, since there are 480 grains in an ounce, there are $\frac{10(640)648}{480\ \pi}$ in the marked off 4,000,000 cubic inches space; and in 50 coffers of air there are $\frac{285 \times 27}{\pi}$ ounces, etc.

With but one other reference to the numerical beauties of this rectified system of metrology, we shall close an otherwise endless topic.

Since in a cubic inch of water there are, at standard circumstances, $\frac{15{,}360}{57}$ grains, and since in an equal volume of air there are $\frac{8}{5}\left(\frac{648}{1{,}000\ \pi}\right)$ grains, and since the specific gravity of water is unity, it follows that the specific gravity of standard atmospheric air must be exactly expressed by $\frac{8}{5}\ \frac{648}{1{,}000\ \pi} \div \frac{15{,}360}{57} = \frac{4{,}617}{1{,}200{,}000\ \pi} = .001224 +$, etc.

From the foregoing brief survey of some few of the numerical beauties of the system, and their direct earth reference, both as to general and particular specific gravity, etc., it seems to be an inevitable conclusion, that in thus unlocking the standard observatories of Egypt and Jerusalem, and building one for the world to-day, we have discovered the key by means of which we may reasonably also hope to penetrate into the sanctum sanctorum of every human science; for to have entered into the holiest place of the universal science is to be at once within those of all its various branches.

APPENDIX A.

"THE SACRED CUBIT."

"And Aaron cast down his rod before Pharaoh, and before his servants, and it became a serpent. Then Pharaoh also called the wise men and the sorcerers: now the magicians of Egypt, they also did in like manner with their enchantments. For they cast down every man his rod, and they became serpents: *but Aaron's rod swallowed up their rods.*"—EXODUS vii. 10-12.

"THE SACRED CUBIT."

"Μέτρον ἀνθρώπου ὅ ἐστιν ἀγγέλου." — Rev. xxi. 17.

In the study of the Great Pyramid, the clearest intellect may find its measure of capacity, and at the end fall short of comprehending all the secrets it contains. It is a beautiful and fascinating study; and, the deeper one investigates it, the more convinced will he become of the master masonry that planned and realized its grand proportions. Its stones cry out in silence far more eloquent than words, its lines and angles speak in language plainer than a written character, against the man who is weak enough to find in such a structure simply accident and not design. If such there be whose inner heart is not amazed with admiration at such a building, — raised, remember, long before the dawn of science, and confounding even modern science, — then let him know, that, in the chambers of *his* soul, there can be no lines that square and harmonize in pyramidic proportions with the heavens and earth.

Note. — I am induced, at the earnest solicitation of friends, to whom I have shown my diagrams, to publish the following article as an appendix to this discussion. The matter treated belongs naturally to the subject of "linear measure," as based upon the *unit* earth-commensurable INCH, and upon twenty-five such inches for a grand standard. This is the "sacred cubit;" and its discussion brings into linear measure as much of the new and important as those who shall have followed us in capacity and weight measure will have seen the latter, in their relations to this cubit, to possess. But, in thus publishing the following in its present shape, I must apologize for its colloquial style. It was written as the third of a series of lectures upon my pyramid labors, and asked for by the officers stationed with me at the Presidio of San Francisco in 1880. It was also intended to deliver the same series before the "Mechanics' Institute" in San Francisco. A sudden campaign, and a subsequent change of station to the East, interfered; and the MS. has remained in its present form ever since. There is now no time for recasting the matter into a style corresponding with the body of the discussion; so, with an apology, it is appended, in the trust that the interesting considerations it reviews will lend as much to the all-important subdivision of linear measure as the subject merits. It throws such a glare upon the "sacred cubit," in its mathematical and geometric relations to the requirements of a perfect linear standard for modern metrology, that we cannot but accord to its lofty and ancient origin a wisdom truly superhuman.

One of the greatest difficulties experienced by the student of the pyramid, is that of systematizing the information that has already been acquired. He experiences this difficulty at the outset, and it increases as he penetrates into its higher and more interior mysteries. The perpetual recurrence of such important sequences of numbers as those which express the "circummetric ratio" and the "solar ratio," 3.14159, etc., 365.242242, etc., and a multitude of dependent series, plunges one into a sea of confusion. No sooner is the intellect focussed upon any part of this mysterious structure, than from the most diverse lengths, angles, shapes, and groupings, the same familiar metric rhythm flashes forth. Now in cubits, then in inches; here in circles, there in squares; sometimes in area, at others in volume,—often where looked for, and more frequently where least expected,—they are ever present. The mind cannot resist the inevitable conclusion, that a most consummate plan regulates the whole design, and links each part to every other and the whole. Seeking for the thread to guide him through a labyrinth of scientific architecture, so worthy of the fabled land it long has shadowed, the bewildered mind a thousand times is led astray. At last *the idea* seems to be within his grasp. A vision of the system shines athwart his gaze, and for a moment the IDEAL pyramid is before him. It is but the mirage; the vision fades; the monument is yet beneath his horizon. Sand, Sahara, and infinity still surround him.

But he does discover, here and there, a green oasis; this he names, an island, merely, in the trackless desert: for his map is necessarily incoherent, from its lack of cardinal bearings.

Such, for years, was my own experience in this engrossing study. The mirage that lures the student on is but a picture, mirrored on the moister atmospheres which bathe these grateful spots of green. But the existence of the image proves the reality of the object, beyond, indeed, our plane of vision, but, nevertheless, real and tangible.

I have already alluded to one of the skeleton systems, which I called "the pyramid method," and which enabled us to connect the coffer with the king's chamber, which contains it, and both with the monument itself. But there the analogy seemed to cease; though it does, however, take a most interesting step beyond. Let us re-examine it, briefly, and in a new form.

$2\pi R$ is the modern symbol for the circumference of any circle in which π is the circummetric ratio, and R the radius of the circle under consideration at the time.

Now, a new, and perhaps the most interesting, form in which we can apply *the pyramid method*, is as follows. You will be kind enough to notice that I use the form 2π into R throughout this application.

(1) In the pyramid, as a whole, 2π into H (its height) equals the perimeter of its base, or equals $4L$, since it is a square base, and all four sides are the same. (See Figs. 3 and 10.)

(2) In the king's chamber, 2π into B (its breadth) equals the perimeter of its side; i.e., equals $2(L + H)$. You will also remember that the length of this chamber is just double its breadth. We have, therefore, the same principles, though with a marked variation in the shape of the geometrical figure to which it is applied.

(3) In the coffer, 2π into H (its height) is again equal to the perimeter of the base; i.e., equals $2(L + B)$. But, in regard to this latter figure, you will also remember that *none* of the dimensions are apparently in any other way related to each other: hence we have the repetition of the principles, with absolute dissimilarity of the ruling dimensions, introduced as a geometric feature.

Let us now repair to the ante-chamber. Upon the granite leaf that stretches across this narrow room is "the boss," the little, raised, and rather more than semi-circular, protuberance which has been described as "the only ornament" in the pyramid. We shall study this very closely later on. At present I shall only allude to the principles under discussion, and which we have just applied to the dimensions of the pyramid, king's chamber, and coffer. This principle re-occurs here in its most ideal simplicity. The boss is $2\pi R$ itself; for 2π, into its own radius, is equal to the indicated circumference of its base. But you will say, that, in all the other examples, I have introduced the element of height in one or the other term of the equation. Why is it not done here, to keep up the analogy? I answer, it *is* there — a silent factor, since it is "unity" itself. The height of the boss is just one inch, or "unity:" it therefore disappears. And so this factor disappeared in each of the other three cases; for, in all those cases, 2π, by itself, is the circumference of the "unit radius:" and the applied formula in every case is as follows: —

The unit circumference (2π) $\times$ the unit (1) radius $\times$ a special radius (height or breadth), yields the other dimensions in perimeter.

But the "pyramid idea" is not to be unfolded by this principle alone. Its help enables us to articulate only a few bones of the mammoth skeleton before us. Let us, therefore, proceed in another direction.

We will return to John Taylor's first discovery, that the circumference described with the height is equal to the perimeter of the square base of the pyramid. In studying this and its kindred facts, at the opening of our last lecture, we were enabled to discover a magistral line, 10303.30 inches in length, which, squared and circled, formed a connecting line between four separate diagrams.

(1) The square of this line gave us the circle of the height of the pyramid. (See Fig. 6.)

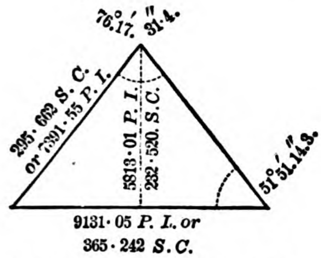
(2) The circumference of such a circle gave us the perimeter of its base. (See Fig. 3.)

(3) The circle of this line gave us the area of the base of the pyramid. (See Fig. 5.)

(4) The perimeter of this base gave us the circumference of the height. (See Fig. 3.)

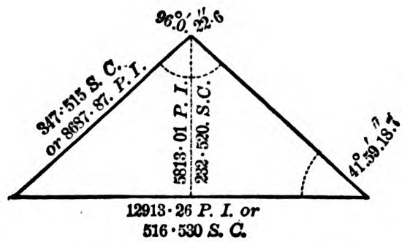
(5, and finally.) The square of half this line is equal to the meridian section of the pyramid, and also to the area of the circle whose diameter is the height of the pyramid. (See Figs. 7 to 9.)

What an intricate confusion! Here are four figures, involving squares and circles, radii and sides, areas, perimeters, circumferences, and other intimate equalities; and yet their simultaneous comprehension is well-nigh unthinkable. It puzzled me for two years. I could not resist returning to it again and again; but, so soon as one set of relations was fixed in the mind, the passage to the next was a process of obliteration.



DIRECT VERTICAL SECTION OF
GREAT PYRAMID

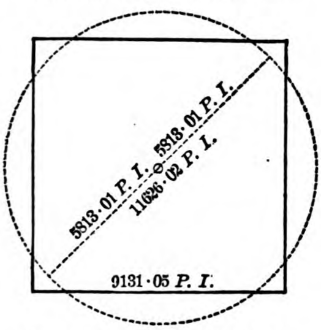
Fig. 1.



DIAGONAL VERTICAL SECTION OF
GREAT PYRAMID

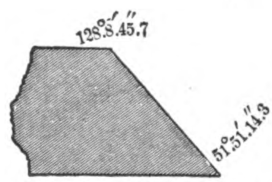
Fig. 2.

EQUALITY OF BOUNDARIES



Great Pyramid's square base,
and circle with radius=Pyr's Vert'l height

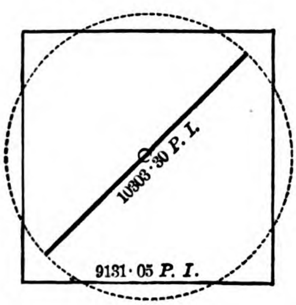
Fig. 3.



π ANGLES OF CASING STONES OF
GREAT PYRAMID;
As affected by its external slope
and horizontal masonry courses.
π = 3:14159 26535 + &c.
= *log.* 0·49714. 98726+&c.

Fig. 4.

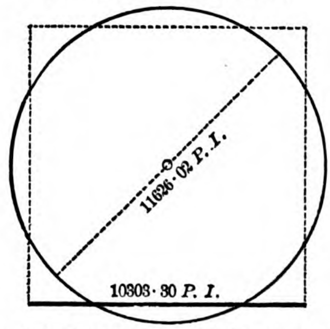
EQUALITY OF AREAS No. 1.



Area of square base of Great Pyramid =
= area of a Circle whose diameter is given
÷ 100 in the Ante-chamber.
P. I.= PYRAMID INCHES.

Fig. 5.

EQUALITY OF AREAS No.2.



Area of circle with G.Pyr's height for radius=
= Area of square whose length of side is given
÷100 in the Ante-chamber.
S. C.=SACRED CUBIT.

Fig. 6.

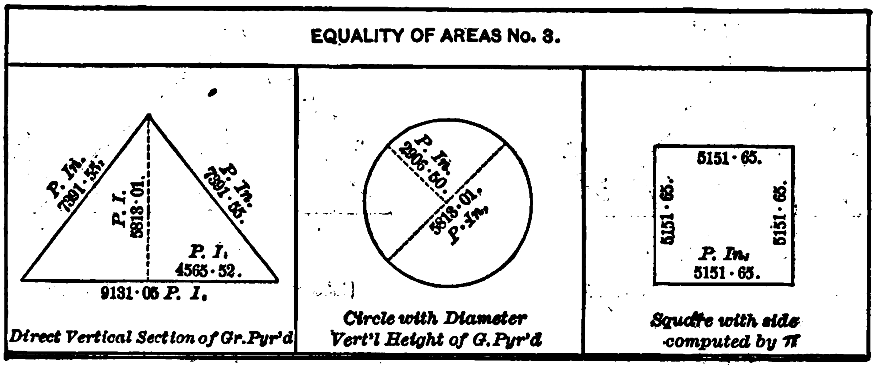
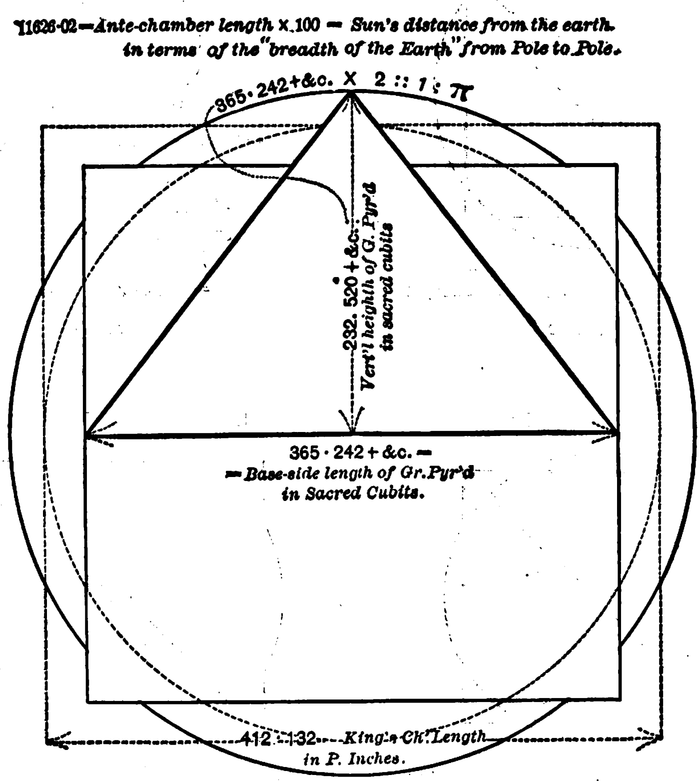


Fig. 7. *Fig. 8.* *Fig. 9.*



EQUATION OF BOUNDARIES AND AREAS,

CIRCLES AND SQUARES, INCHES INSIDE AND SACRED CUBITS

OUTSIDE GREAT PYRAMID

Fig. 10.

At last the solution occurred to me in the form of a single geometric figure, which I shall term, *par excellence, the pyramid diagram.* Its discovery was prolific with attendant ones, some of which I hope to review this evening.

THE IDEAL DIAGRAM.

The pyramid diagram may be put under two forms,—the one absolute, and the other approximate. I shall call the former the architect's ideal, the latter a workman's diagram.

Let us proceed to the examination of the first, or ideal geometric diagram.

Let *ABC* represent the meridian section of the Great Pyramid.

Complete the square *ABFG.* It is equal to the square base of the pyramid.

With the vertex *C* as a centre, and a radius *CD* equal to the height of the pyramid, describe the circle *DIH.*

By John Taylor's discovery, its circumference is equal to the perimeter of the square *ABFG* base of the pyramid.

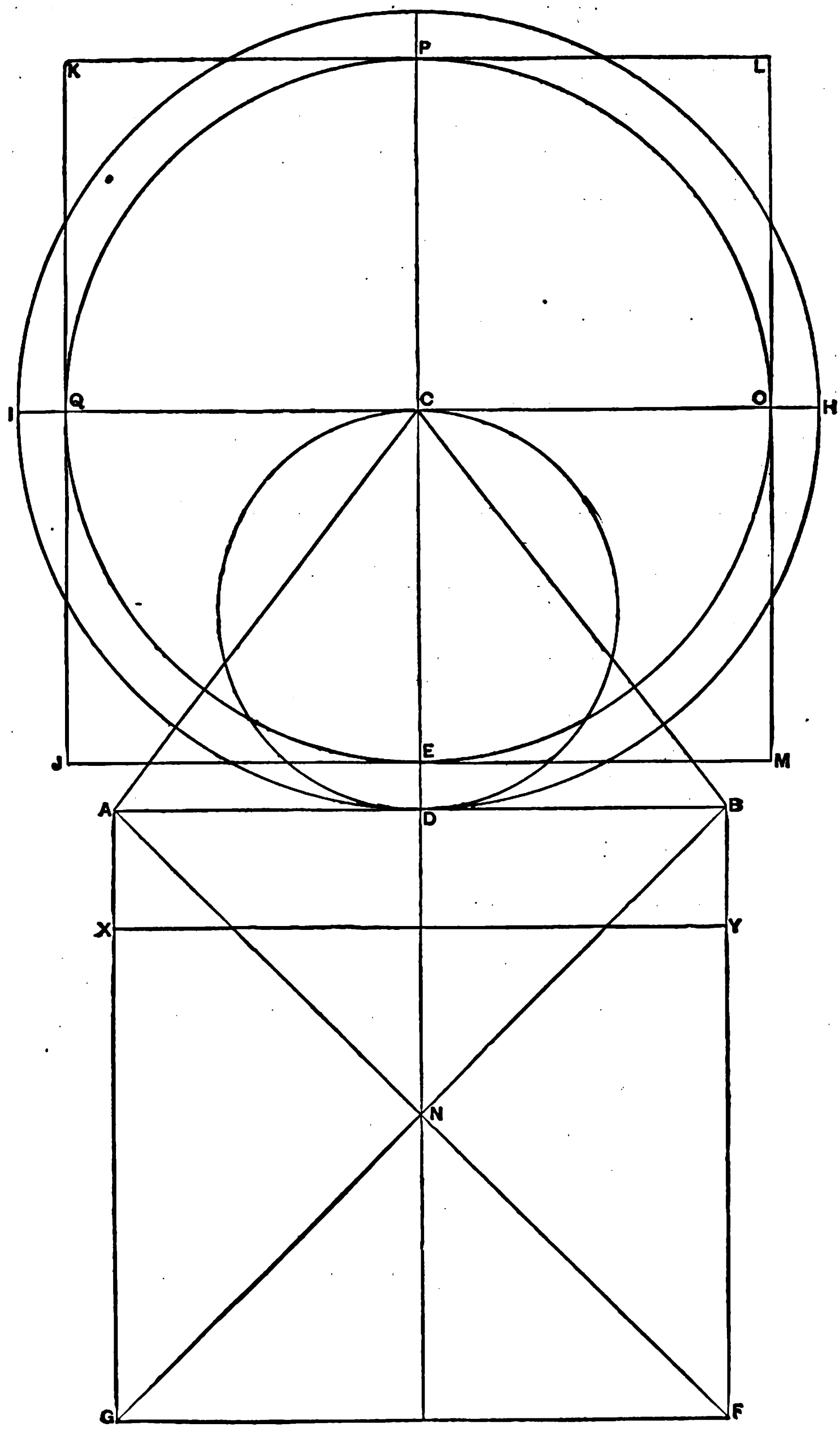
By means of the modern circummetric ratio (3.141592, etc.), determine a square *JMLK,* whose area is equal to the circle of the height.

Inscribe in this square another circle *EOPQ.*

Then will the area of this new circle be equal to that of the square base of the pyramid itself.

Finally, upon *CD* as a diameter, construct another circle: its area will be equal to that of the meridian section of the pyramid, and to one quarter of the square *JMLK.*

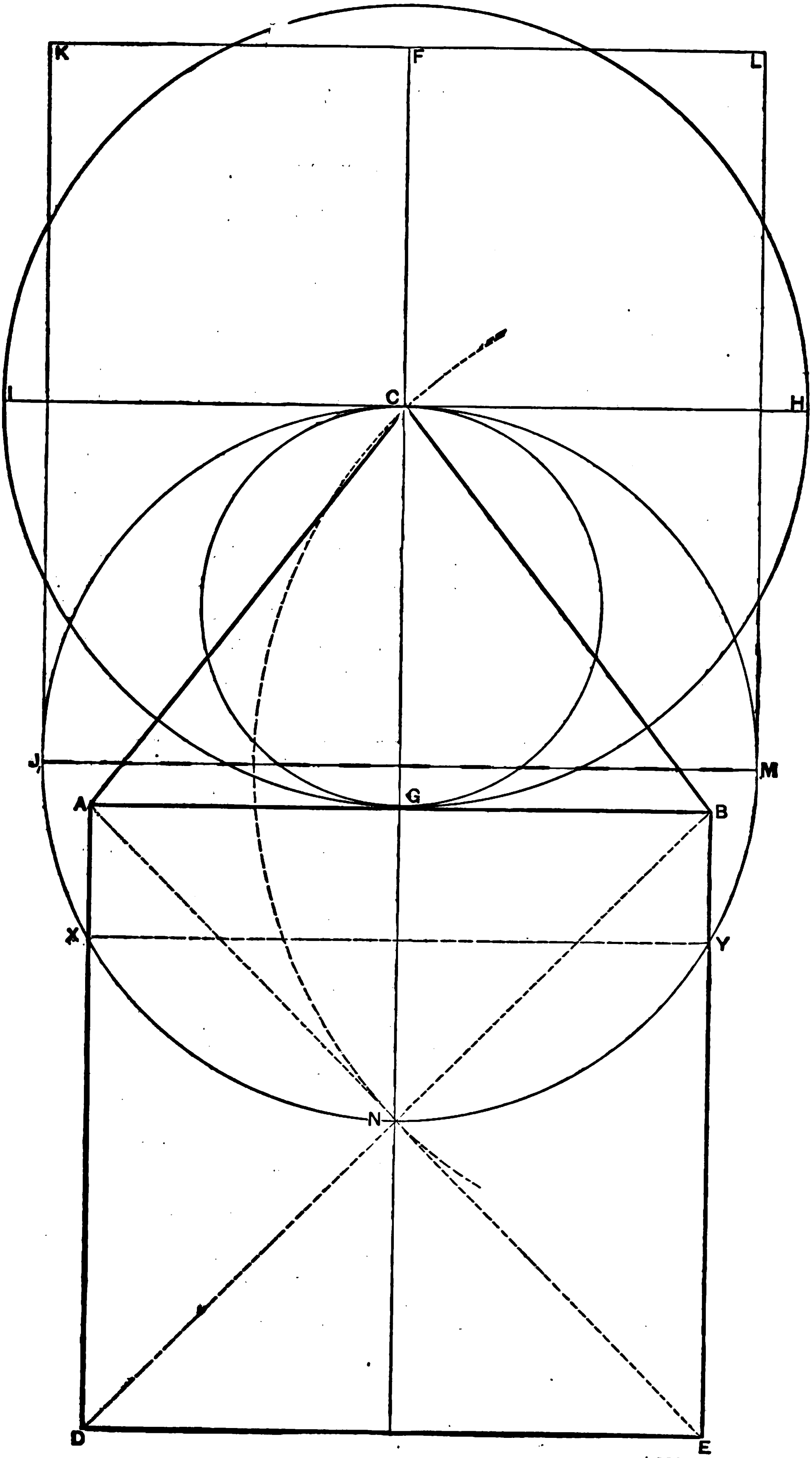
This most remarkable sequence of squares and circles, depending upon the triangular dimensions of the pyramid's meridian section, not only unites into one, easily comprehended, diagram, all the confusing relations we have just noticed, but, as we shall see later on, is a key to one of the most universal geometric solutions that has ever been discovered.



THE PYRAMID DIAGRAM.

(ARCHITECT'S IDEAL.)

Fig. 11.



THE PYRAMID DIAGRAM. (WORKMAN'S METHOD.) Fig. 12

THE WORKMAN'S DIAGRAM.

Let us turn now to the approximate or workman's diagram. This is a geometric construction without the use of π.[1]

As before, *ABC* is the meridian section of the pyramid, and *ABED* a square equal to its base.

Join *C* and *N*, the vertex of the pyramid, and the centre of the square, and upon it as a vertical diameter construct the circle *NJCM*. Then will the area of this circle be equal to that of the square base of the pyramid.

Upon its horizontal diameter *JM*, construct the square *JKLM*. The area of this square will equal that of the circle of the height of the pyramid.

With *CG*, this height, as a radius, and *C* as a centre, describe this circle of the height: then, as we already know, its circumference equals the perimeter of the pyramid's base. Upon *CG* as a diameter construct a circle. Its area, as before, will be equal to that of the meridian section of the pyramid, and also to a quarter of $\overline{JM}^2$.

---

[1] Mr. Charles Latimer has shown, in vol. i. of the International Standard (p. 29), that the identical triangle which forms the key to the architecture of the Great Pyramid, may be developed from the circummetric relations of the square whose side is $81 \times 100 = 9^2 \times 10^2$. The diagram he gives comprises all the dimensions of the skeleton. In discussing this diagram upon p. 245 of the same volume, I have myself called attention to the fact, that had Mr. Latimer continued his series of squares, circles, and triangles downward a degree farther, he would have arrived at the very ones employed by Mr. Parker in his interesting essay upon the Quadrature of the Circle (John Wiley & Sons, publishers). Mr. Parker's method of arriving at the relation of circles to squares, is an entirely original one, and is based upon a rigid discussion of pure geometric form, and of abstract number. As a result of this discussion, he arrives at the remarkable fraction $\frac{20612}{6561}$, as expressive of the *natural* circummetric ratio. In his deductions, Mr. Parker does not, in the remotest degree, touch upon, or employ, any of the methods made use of by modern geometers in deducing the π value. He arrives at his ratio by a direct examination of the "opposite duplicate ratio" $\left[\sqrt{\sqrt{3}} : ((3)^2)^2\right]$ of equal equilateral triangles and circles to their mutually standard unit square. It is noticeable that the Parker modulus is having considerable attention turned to it in higher scientific circles to-day, after having been received with a cold shoulder some score of years. Now, as Mr. Latimer has shown the relations of the pyramid triangle to the square of 81 units, and as I have shown the relations of the Latimer diagram to the Parker modulus, the relations of the latter to the pyramid triangle are logically established. For all practical purposes, therefore, the "workman" may employ the Parker modulus in constructing pyramid triangles. Whether the latter be absolutely true, as claimed by Mr. Parker, or merely a remarkably close approximation, it is, at any rate, accurate beyond any possible degree of detection, even for so stupendous a structure as the Great Pyramid of Gizeh. Hence, representing by H the height of a pyramid triangle, and by S its base, the workman may employ the formula $10306H = 6561S$, to determine the height required for any given base; or, conversely, the base for any given height.

In order to demonstrate more clearly to your minds the simplicity with which this geometric figure clears away the inevitable confusion surrounding the four separate diagrams, by means of which these relations have hitherto been expressed, I have represented the workman's diagram again, and indicated its more important lines by their familiar numerical values in inches.[1]

We have a very important use to make of the pyramid diagram this evening. I hope to show that it is the mystic problem, upon the pyramid's masonic trestle-board, which fits as beautifully into the interior chambers and entire furniture of the edifice, as we have now seen that it certainly does into the building as a whole.

There are, however, one or two reflections which I desire to make while this diagram is fresh in our minds.

The first is this: the pyramid diagram establishes beyond a doubt the *possibility* of squaring the circle geometrically. Mathematicians have for ages been contented with approximations, and have acquiesced in the interdiction placed upon this problem by the learned. The French Academy and the Royal Society of Great Britain, within our own century, have passed resolutions not to entertain any demonstration looking towards this greatly desired solution. Convinced that the demonstrations of Playfair and Legendre are correct, and that π has no simpler expression than its infinite decimal form, they repudiate every effort to the contrary, and refuse even to listen to those students who are yet unconvinced. The result is, that he who labors at this problem is regarded as either ignorant or of unbalanced mind; and the great problem of squaring the circle is thus in utter disrepute. For one, I am not convinced: indeed, I am satisfied to the contrary. I will not make the rash assertion here, that I can solve this problem; that is not our theme to-night: but I will say, and I think the pyramid problem fully establishes it as a fact, that such *geometric* relations certainly do exist between squares and circles, in nature, that there must be, and is, a geometric means of passing accurately to and fro from either to the other in all their parts. The study of the

[1] Diagram omitted. For dimensions, see Figs. 1 to 10, etc., which may be written on corresponding lines of Fig. 12, and the rest calculated.

Great Pyramid from the mathematical stand-point convinces the student, that no problem was paramount to this one in the mind of its architect. It is unreasonable to expect the work of even the wisest of men to be void of error; man is not even a demi-god; and his work must fall short of his theory, even if the latter be correct; hence we must expect, even in this great metric monument, only enough of realization to suggest, at length, to some lucky and painstaking student, the ideal aimed at by its mortal builders.

The second consideration is this: that he who discovers a geometric method of constructing a triangle similar to that of the theoretical meridian section of this Great Pyramid, will have squared the circle. For given a triangle geometrically constructed, whose base shall be 9131 +, etc., and whose height shall be 5813 +, etc., to any degree of accuracy that we may be able to measure them, and the pyramid diagram follows. The circle will then be squared and verified; for all the squares and circles in this intimate series, based upon that triangle, are related by equalities.

Now, this triangle does exist: it is built into and symbolized throughout the monument. Theoretically, we can renew it to any desired degree of accuracy by using the modern value of π.

The great question, therefore, is, How did the shepherd kings obtain this triangle? By approximations? by a π value? by any numerical expression? or by a strictly geometric construction? My own studies of this intricate subject lead me to the conclusion, that the architect of the Great Pyramid had a simple geometrical construction; that he could and did square the circle. But it necessarily follows, that he also had a perfect *numerical* expression for his work; for it is absurd to suppose that correct geometry is inexpressible in numbers. Not necessarily in *decimal* numbers, but in some form or other of numerical expression, this circummetric ratio can certainly be written. The circummetric ratio is no more difficult of expression than the ratio of the perimeter to the diagonal of a square: both are incommensurable, — equally so and similarly so. In the square whose side is one, this latter ratio is $4 : \sqrt{2} = \frac{4}{\sqrt{2}}$ $= 2.82 +$, etc., forever. Nevertheless, $\frac{4}{\sqrt{2}}$ is a perfect arithmetical expression for this incommensurable decimal; and its *geometric* relation — a diagonal within a square — is shown on every draughtsman's trestle-board. Number is the *soul* of form or geometry, and every mathematic line must have its "perfect" counterpart in number.

The shepherds, you remember, were a mysterious people. They appeared suddenly in Egypt: they came with a purpose, — to build the pyramid, at a spot elected from the whole surface of the earth by its unique and solitary fitness. Their work completed, they departed as suddenly as they had come. We know that they next settled in Palestine: we know that their descendants thereafter built no pyramids. Hiram, — king of Tyre, — and Hiram Abiff, were descendants of these shepherd kings. A later race of shepherds, the Israelites themselves, were also related to the shepherd kings; and they in turn followed in the footsteps of their more ancient predecessors, whom, the Bible tells us, the Lord, in former days, had also led up out of Caphtor, or of Egypt. Solomon, Hiram, — king of Tyre, — and Hiram Abiff, were the master-masons of the only other symbolic building that these peoples were connected with; and, though its style was so utterly dissimilar, yet it is reasonable to suppose that perhaps they both belonged to the same *school* of architecture.

Although enough has been already adduced in the body of the work to establish this connection upon a positive basis, we shall allude, in passing, to the following: —

THE PYRAMID AND SOLOMON'S TEMPLE.

The temple of Solomon is destroyed, but its measures and dimensions are sacredly preserved. Freemasonry is familiar with them; and the great book of symbols, the Bible, carefully records them.

Some years ago (1878) Major H. A. Tracy, R.A., in a published article entitled, "Was the Architect of the Great Pyramid inspired?" called special attention to the temple of Solomon, and pointed out some remarkable mathematical relations between its dimensions, which stamped it as undoubtedly pyramidic. Taking its interior dimensions, after its finished casing of fine cedar, he studied particularly the diagonals of the various grand divisions of the temple. Now, every builder knows that diagonals are of primary importance in verifying the perfection of rectilinear work. Without going into detail, I will say, that, among other peculiar relations, Major Tracy discovered that

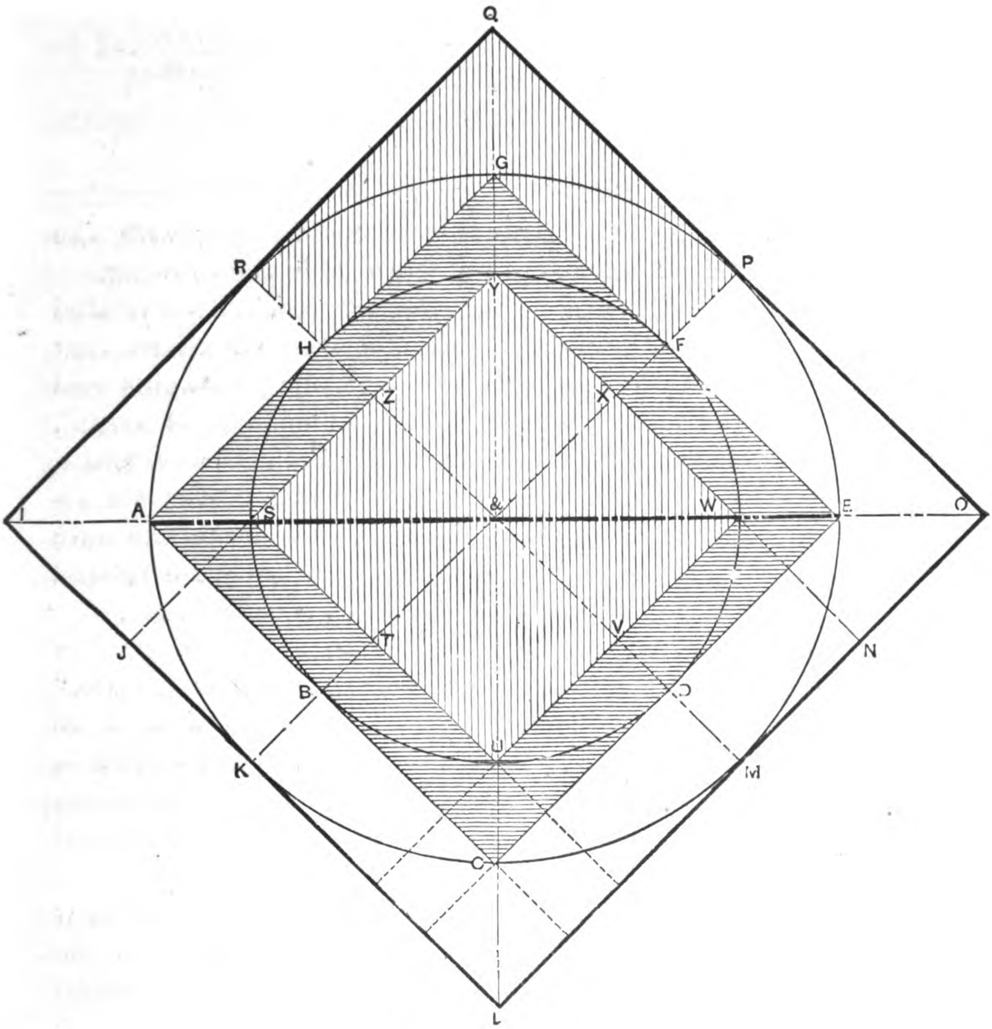
The diagonal of the floor of the porch	$= 10\sqrt{5}$ cubits; i.e., sacred cubits.
The diagonal of the floor of the holy place	$= 10\sqrt{20}$ cubits; i.e., sacred cubits.
The diagonal of the side-walls of the holy place	$= 10\sqrt{25}$ cubits; i.e., sacred cubits.
The diagonal of the floor of the Holy of holies	$= 100\sqrt{50}$ inches, etc.

The latter expression, $100\sqrt{50}$, is $\pi \times 9$ to thousandths of accuracy. These numeric values are all intimately connected with the pyramid system of construction. To mention, in passing, but one, it may be stated that $50''\sqrt{5} = 111''.803398+$ is the height of the western wainscot in the ante-chamber, a room, that, as we shall see later on, is filled with similar intimacies. They are also peculiarly suggestive of Mr. Simpson's treatment of the diagonals in the king's chamber of the Great Pyramid, and which were alluded to in our last lecture. In both of these buildings, these concealed and rhythmic values yield only to a sacred cubit of 25 earth-commensuric inches, or to the single "unit inch" itself.

While studying these relations of Major Tracy, pencil in hand, in 1878, and long before my discovery of the pyramid diagram, I noticed that the proportions between the main parts of the temple of Solomon could be expressed in a diagram that involved the relations of squares and circles of the very simplest order. The relations between the squares and circles in the *pyramid* diagram are, upon the other hand, of the most complex order. The two sets of relations are, in fact, as diverse from each other as possible. This will be more manifest when the temple diagram shall have been explained.

The discovery of the temple diagram, in fact, preceded that of the pyramid by at least four years, and, in reality, led to the latter. Let us examine it.

Draw the line *AE* equal to 1,000″, and upon it, as a diameter, construct the circle *AKCMEPGR*. Its circumference will be equal to 3,141.59+″, etc., and its area 785,397.5+ square inches, etc., both of which series of numbers are of the utmost importance to mathematicians.



THE TEMPLE DIAGRAM.

(SOLOMON'S.)

Fig. 13.

Upon the same line *AE*, as a diagonal, construct a square *ACEG*. It will be inscribed in the primary circle, and its side be equal to 707.1067″, etc. This, too, is a fundamental multiplier in geometrical calculations.

Inscribe in this square another circle, *SBUDWFYH*, and in this latter circle inscribe, in turn, another square, *SUWY*.

Finally, circumscribe about the whole figure the square *IJKLMNOPQR*.

Draw diameters to the original circle, *AKCMEPGR*, passing through the successive 45° points, and commencing with the primary line *AE*, and complete the figure as in the drawing.

It is the temple diagram. Its circles and squares are related to each other by the simplest of geometrical constructions, that of inscription and circumscription. Proceeding outwards and inwards from the square *ACEG*, whose diagonal *AE* is 1,000″, we have a circle and a square circumscribed, and a circle and a square inscribed. The successive squares and circles are related to each other, as to areas, by *doubles*, and the alternate ones by quadruples; and the various parts of each are related by a most interesting and intricate series of equalities, etc.

So, too, the perimeters and circumferences are related by notable proportions and multiples of the "circummetric ratio." To mention but one circumferential reference, in addition to those already noted, the perimeter of the original square is 900π to within $.99\frac{1}{3}''$ of accuracy. This is a general principle, and is true of all circles; though I have never before seen it stated or used.

The error is so small, that, with no attempt at correction, it amounts to but $22\frac{1}{2}$ inches in a mile of circumference. In the whole circumference of the earth, it is only 8.8 miles, or about $\frac{1}{400}$ of the diameter. In other words, to bring the matter down to a plane of comparison with ordinary handwork, the circumference of a 10-inch circle measured by this method would err by only .011 of an inch. This is very small when we remember that our finest standard measures are only graduated to hundredths, and that a thousandth of an inch is absolutely invisible — in the sense of indistinguishable — to the naked eye.

I am digressing for the moment; but my excuse is this, — that we have here, thus indicated in the temple diagram, one of the closest working approximations to the "square of the circle" that I am aware of. I shall call it Solomon's "temple-rule;" and, without occupying your time with the steps by which I have succeeded in putting it into a beautifully simple and most accurate form, it may be stated as follows:—

In any square, 1,000 perimeters diminished by a correction of one diagonal is equal to 900 circumferences of the circumscribed circle. You will notice the round numbers which enter into this statement, and also that the diameter of the circle is, of course, the diagonal of the square. Now, the error in this formula,

$$1000\,P - D = 900\,C,$$

is only .00000709. In the whole 25,000 mile circumference of the earth, it amounts to only .175 of a mile (or to about 900 feet), which is far closer than modern science is yet certain of that distance.

By the temple-rule, the value of π may also be put under a very simple form,

$$\pi = \frac{2000\sqrt{2} - 1}{900},$$

whose decimal value is 3.1415856, or practically 3.14159, closer by unity in the place beyond, than the modern working-rule of 3.1416.

"THE TEMPLE-RULE."

1000 perimeters of any square — 1 diam. = 900 circumferences inscribed.

To verify this formula, let us apply it to the square whose diagonal is 1,000. Since the diagonal of every square is equal to a side into the square root of two, we have the following:—

$$D = S\sqrt{2}$$

$$\therefore\ D^2 = 2S^2,$$

$$S^2 = \tfrac{1}{2}D^2,$$

or

$$S = \sqrt{\frac{D^2}{2}}.$$

Substituting for D^2 its value, 1,000,000, we have

$$S = \sqrt{500000} = 707.10678 +, \text{ etc.},$$

$$\therefore P = 4S = 2828.42712 +, \text{ etc.}$$

Hence, substituting, in the temple formula, the above values of P and D, we have, as follows:—

$$1000\,P - D = 900\,C,$$

or

$$2828427.12 - 1000 = 900\,C,$$

or

$$2827427.12 = 900\,C,$$

$$C \therefore = 3141.585 +, \quad \text{say } 3141.59.$$

Now, it is manifest that no use, in the above calculation, has been made of the modern value π; and yet it is equally clear that the result is correct to the hundredths, and this is a circle whose diameter itself is 1,000.

The modern π value for the circumference is $2\pi R$; or as R in this case is 500,

$$C = 1000\,\pi = 3.141592 +, \text{ etc.}, \times 1000 = 3141.592 +, \text{ etc.}$$

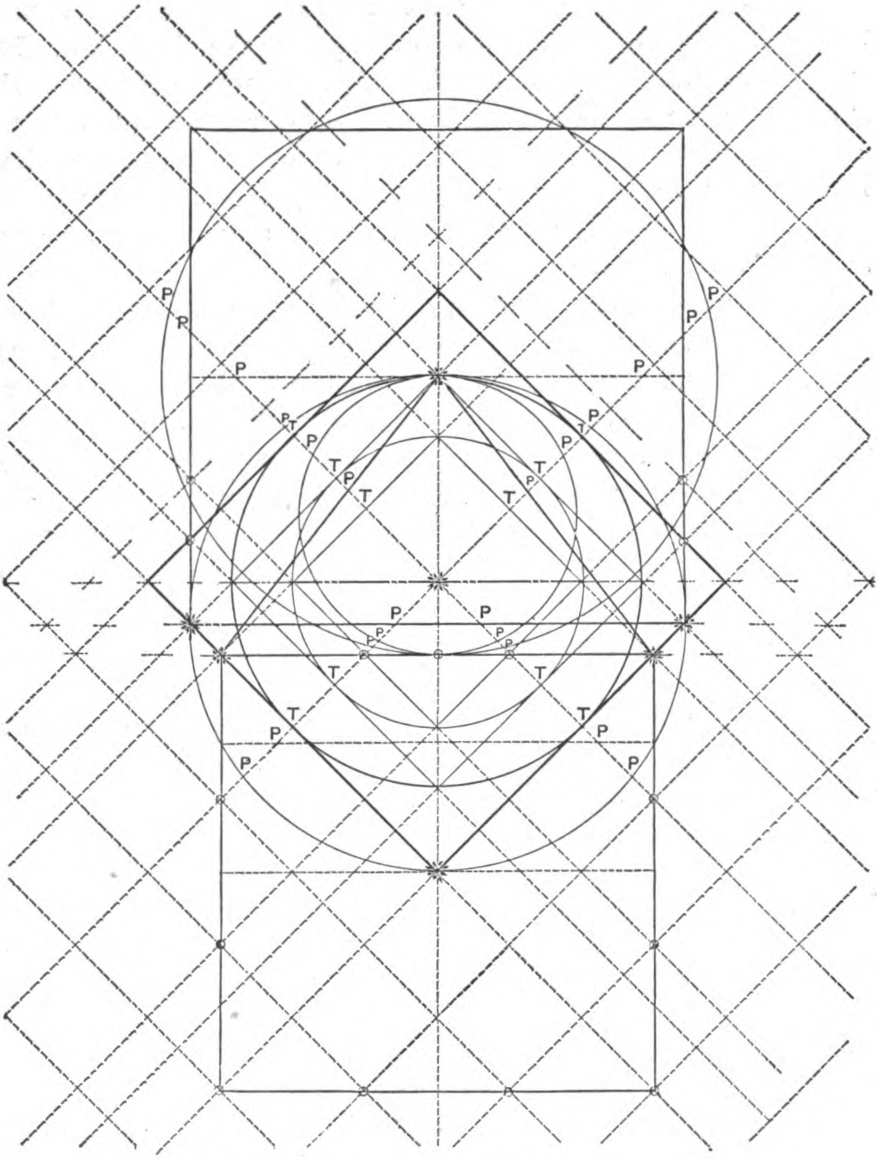
The formula is, of course, equally true for any other value of D, and, if it is desirable, may be developed to almost any degree of accuracy; but, as a working-formula, its simplicity, both for memorizing and application, is as noticeable as its extreme accuracy.

But I have not yet explained why I have termed this geometrical figure the temple diagram. There are several reasons. The principal one is this: in a figure so constructed upon a base-line of 1,000 sacred inches (or twenty-fifths of the sacred cubit), the area of *FPQRHGF* equals that of the "porch" of Solomon's temple (125,000 square inches); the area of the original square *ACEG* equals that of the holy place (500,000 square inches); and the area of the interior square *SUWY* equals that of the Holy of holies. It is to be noticed here, in addition, that this interior square, or Holy of holies, is inscribed in a nest of circles and squares whose original radius, 500″, was the side-length of the Holy of holies itself. Thus having set out with a given line as a radius, we have here arrived at the same line employed as the side of an inscribed square. From the Masonic stand-point the tesselated pavement of Solomon's temple is of great symbolic import; and an examination of the temple diagram develops the fact that its whole groundwork is a simple checker-board 250 inches, or 10 cubits, on a side, or each square of which contains an area of 100 square cubits. Nor is the beautiful border, of smaller squares and rectangles, to be overlooked in the mathematical relations of this ancient and still wonderfully symbolic pavement.

But we must draw this examination of the temple to a close. It must certainly now be clearly manifest, that these two remarkable buildings belong to the same school of architecture. While constructively they are entirely dissimilar, they are, nevertheless, intimately related in spirit. It was to the temple diagram that I resorted in my search for the pyramid triangle. I reasoned that it must be hidden there somewhere. Study soon demonstrated that the two diagrams were indeed most strangely related, as though, in fact, they were an involute and evolute. Drawing the working-diagrams to the same scale, I found many points of coincidence, and at last proceeded as follows:—

Producing the sides *YS* and *YW* of the Holy of holies, they intersected those of the outer square in the points *J* and *N*; and, joining the points *G*, *J*, and *N*, I obtained an approximate *workman's* "pyramid triangle."

I shall not take you through the maze of calculations that followed, but, to establish the connection thus discovered between the diagrams and buildings, merely ask you to examine the combined figure, in which I have superfixed the pyramid diagram upon that of the temple. This is merely constructed upon the "workman's" or approximate plan, but the numerous coincidences throughout the diagrams cannot fail to establish their intimate connection. The harmony between the two diagrams is such that every geometrical line and important point of the one is directed, as it were, or located, by those of the other. Thus, in our combined diagram, we have marked with a **o** the commonly important points of both figures, and by a * the seven points of special coincidence. If built upon the π relations, the coincidences of these diagrams are approximations



THE PYRAMID AND TEMPLE DIAGRAMS HARMONIZED.

(APPROXIMATE.)

Fig. 14.

of the closest possible character,—approximations than which none closer can be realized in the whole range of geometry. If, however, the pyramid be constructed upon the "extreme and mean ratio," as Ballard opines, the relations will possibly come out still closer,[1] nor do I doubt but that between the architect's ideal diagram and that of the temple, there may yet be found an exact connection.

THE DIAGRAM IN THE KING'S CHAMBER.

Let us now retrace our steps through the great monument of Egypt, and, retaining in our hands its complicated diagram as a *gauge*, remeasure its dimensions.

If it *be* a sacred shrine, let us obey the prophetic mandate of the angel to the holy St. John, and "arise and measure the temple and its altar."

If perchance we have indeed discovered *a key* to the mystic combinations that have for so many centuries defied admission, we may then prepare ourselves for some surprising revelations of symmetry and design.

Let us repair immediately to the best known, or king's, chamber. I have already alluded to the peculiar adjustment of its floor to the lower tier of its wall-blocks, whereby the contents of this portion of the room is made to equal 50 times that of the coffer, and whereby the ratio of the ark to the brazen sea in Solomon's temple is exactly realized. This adjustment is accomplished by raising, as it were, the floor of the chamber up into the room some 5 inches. It would never have been discovered but for the partial destruction of this floor in one corner. Here the treasure-seekers of former years have removed some of the floor-blocks, and thus revealed the fact that the side-walls continue downward a few inches beyond the general level of the floor. This adjustment has given rise to what are known as the first, second, and mean heights of the king's chamber. The first height is that of the room itself, or, in *pyramid* inches, is about 230″; the second height is that of the side-walls, laid bare, or about 235″; and the mean of the two is 232.52″.

[1] I am indebted to Mr. Jacob M. Clark of Elizabeth, N.J., for this idea, and shall gratefully leave it for his own labors to establish or contradict.

Using, now, the mean height (232.52″) of the chamber as the altitude of a pyramid triangle, and drawing the pyramid diagram to the resulting scale, we find that its magistral line is 412.13″, or the length of the chamber. The base of the triangle is the year number, or 365.24″; and the radius of the circle, whose area is $\overline{365.24}^2$ (or the base of our new pyramid), is 206.06″, or the width of the chamber. Moreover, the radius of the circle whose area is that of the pyramid triangle, is 116.26″, etc. In fact, the diagram, thus applied, reproduces all the dimensions of the chamber. It must also be manifest, that, with these dimensions, a *model* pyramid may be constructed.

Viewed in the light of this diagram, the king's chamber is indeed an image of the pyramid, and a beautifully proportioned image, too! For further examination shows that the two similar diagrams (that of the pyramid itself, and that of the king's chamber) are related to each other by the cubit number 25 as a ratio. In other words, line for line, dividing every dimension of the original pyramid diagram by 25, we obtain the corresponding one in that of the king's chamber.

There are many other interesting relations between these two diagrams, that, for want of time, I must pass over.

Let us now construct a pyramid triangle whose height shall be the *width* of the king's chamber, and complete the resulting pyramid diagram.[1] We shall thus obtain one in which the *year* number is now the magistral line. The perimeter of the base of the resulting model pyramid is made up of the two other important dimensions of the king's chamber, each one being doubled. These are the length (412.1318″) of the chamber, and this time its remaining or second height (235.2431″).

But since the *magistral line* of this new diagram and the pyramid *base* of the former one are the same (i.e., the year number), the two diagrams have a mutual square whose side is 365.242″, and may thus be made *continuous*, or to form, as it were, a double diagram. The two halves of this double diagram are thus united by a circle and a square, the diameter

[1] We just constructed the diagram upon its height.

and side of which are respectively equal to the all-important number, 365.242.[1]

The upper half of this double diagram refers, then, to the king's chamber by its mean height; the lower half to the same chamber by its second height; and each, or rather both, to the *primary* pyramid diagram itself, as thus doubled by the cubit number 25. Surely this is the perfection of design!

But, moreover, the pyramid diagram fits as beautifully into the dimensions of the *coffer* as it does into those of the king's chamber, in which it is found. So, too, the dimensions of the ante-chamber verify by the same consummate rule.

But, even more wonderful to relate, if we divide the dimensions of the upper half of the double diagram of the king's chamber by 4, and those of the lower half by 5, the ante-chamber diagram results in the one case, and that of the coffer in the other.

The accuracy of these results involves inches to their tens of thousandths. They satisfy the various dimensions of the monument to the last reliable figures of modern measure, and they advance beyond the possibility of stone-cutting and architecture into the regions of pure mathematics of the highest order. To obtain these results, and to verify their intimate relations to the pyramid and to each other, months of the very closest mathematical application have been necessary. But such work repays the student. They convince him of a design whose scope is simply *perfect.* Every application of the pyramid diagram unlocks some new mystery in this wondrous edifice; and each new discovery suggests a chain of others, that runs out almost hopelessly beyond the power of man to master in a lifetime.

THE ARCHITECTURE OF THE ANTE-CHAMBER.

I shall now illustrate how this diagram may be employed to verify the architectural work of such a noble metric monument, and how, perhaps, it may be yet employed by future generations to repair and reconstruct the pyramid thus theoretically renewed.

We will confine our attention to the ante-chamber.

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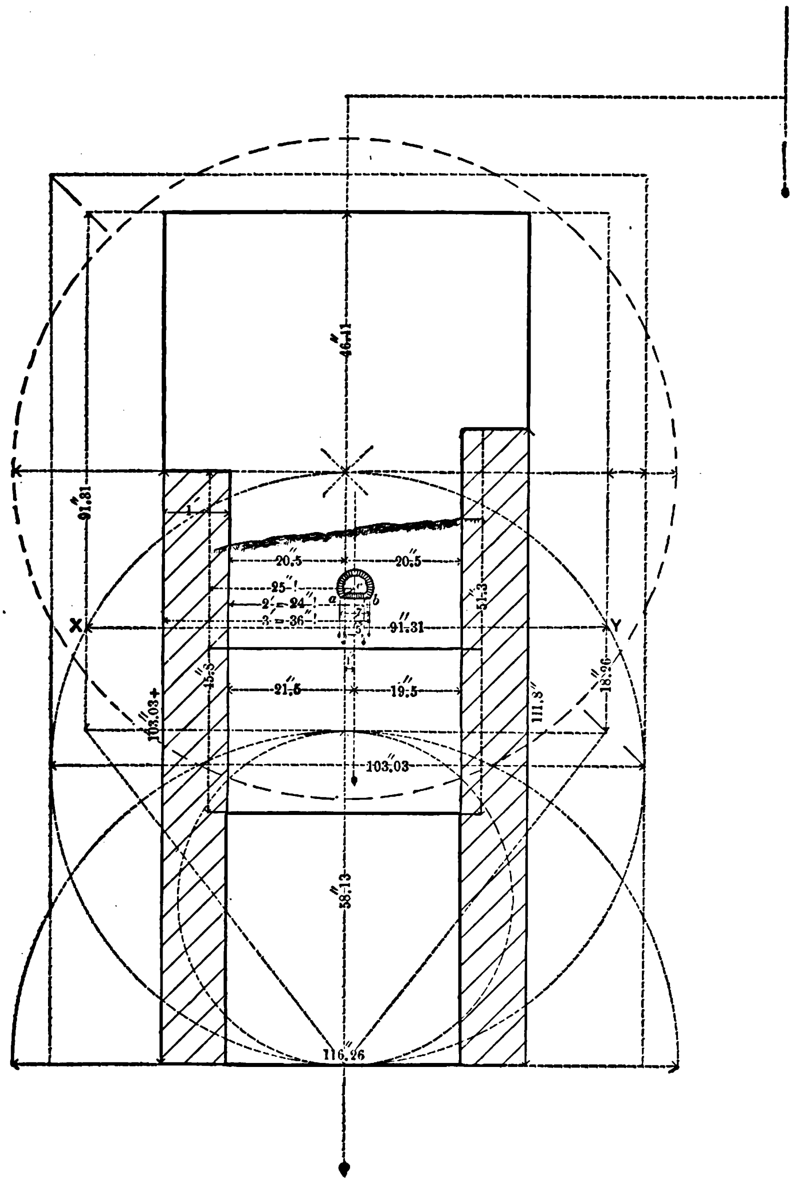
[1] See Fig. 18, and its discussion, for a representation of what is meant by a double diagram; the square *S* and circle *C* having the magistral line 365.242 for their respective side and diameter.

The world to-day stands at the threshold to the ante-chamber of modern civilization. As the Great Pyramid is a masterly Masonic emblem of human experience, so, in the momentous initiation of the race into the mysteries of more perfect life, man is just leaving the great step at the southern end of the Grand Gallery, — a yard high, and a yard + a cubit wide, — and stoops at the narrow passage of Masonic trial he must yet undergo ere he may emerge into the millennium symbolized beyond. The whole world is upon the tiptoe of expectation! We are nearing an awful crisis! The struggle coming, even now at hand, is to be one between ignorance and knowledge, wealth abused and poverty made desperate; between the power of evil and the strength of right. Men run to and fro, and knowledge is increased. The times are rich in signs; and, even as man stoops at the narrow passage-way he yet must traverse, the dread roar of the Cerberus of Nihilism greets his ear, at once from unenlightened Russia and enlightened England.

But the *right* must ultimately prevail. Convinced of this, let us precede him into the symbolic ante-chamber, "and fear no evil." We may already read there, for our encouragement, some of its many teachings.

The figure we shall now examine is a cross-section of the ante-chamber. It is made by a vertical plane passing down east and west just in front of the granite leaf that stretches across this apartment. Upon this plane, you will notice, I have projected the leaf, or standard bar as it is sometimes designated. This bar consists of two pieces of granite. The lower one is carefully squared and finished, as is the upper block, save upon its top surface, which was left by the architect in a purposely irregular condition, as indicated in the figure. These blocks are held in place by two grooves, cut downward from the tops of the two wainscots represented in cross-section. The bottom of the lower block rests upon the bottom of these grooves; while that of the upper one is flushed down upon the one below, so as to form, as it were, a continuous piece of masonry. No cement is used: the separateness of the blocks is thus clearly defined. It is upon the upper block that the boss, already described as the only ornament in the pyramid, occurs. This boss is noticeably displaced from the centre of its block, both



THE PYRAMID DIAGRAM AND A CROSS-SECTION OF THE ANTE-CHAMBER.

(ITS ARCHITECTURE VERIFIED.)

Fig. 15.

as to vertical and horizontal arrangement. You will perceive that the western, or right-hand, wainscot is somewhat higher than the eastern, which is 103.03″ from top to floor: the height of this western wainscot is 111.803″. The thickness of the wainscots is one foot each, and the maximum width of the chamber (over them) is 65.2″. The width between the wainscots, or under the granite leaf, is 41.2″. The length of the chamber is exactly 116.26″, 103.03″ of which is floored with granite, the remaining 13.23″ being of limestone. The above dimensions are all in "pyramid inches."

But such an enumeration of dimensions conveys very little information. I shall give you a far better idea of this chamber by stating that it has circummetric or π proportions. We have already noticed that the pyramid diagram satisfies these dimensions at one-fourth of those of the king's chamber; or, since that is one twenty-fifth of the pyramid itself, we shall explain ourselves still better by calling attention to the fact, that in the diagram, as applied to this chamber, every line comes out exactly one-hundredth of the similar line in the primary pyramid diagram.

To illustrate more clearly the accuracy with which this diagram fits the ante-chamber, I have located it in mathematical lines upon the cross-section. You will notice that it is inverted. The vertex of the pyramid triangle is at the centre of the floor: its base accurately divides the lower block of the granite leaf. The centre of the square base of the model pyramid is at the height of the eastern wainscot, while the ceiling of the chamber is limited by that side of the square which lies opposite to the base of the triangle. The horizontal diameter of the large circle, which limits the floor of the chamber, is also exactly equal to the whole north and south length of this floor, 116.26″; while its granite length of 103.03″ is equal to the height of the eastern wainscot, and to the magistral line of the diagram. Moreover, as though to intensify the coincidence of this diagram with the marked architectural and geometric features of the cross-section, this magistral line of 103.03″ exactly *bi*sects the equal height of the eastern wainscot. The difference, 13.23″, between the length of the whole (116.26″) of the floor of the ante-chamber, and that part (103.03″) which is of granite, is also shown in the diagram to be the difference of the diameters of the two circles, which form such important elements in its sequence.

A longitudinal section (i.e., north and south) of this chamber results in just as beautiful an application of the pyramid diagram.

It is manifest that such a diagram may be used to verify the finish of such a chamber. The process is geometric; it is architectural; it is more, it is *Masonic.* It is one of the grandest diagrams upon the Freemason's trestle-board! It affords a theoretic standard by means of which we may renew this building, or erect another as free from error as instruments can work, and measures test the accuracy of our efforts. It is beyond conception that even boasted modern skill could better the solid granite facts thus built into this remarkable room. In spite of fifty centuries of age, its noble architecture is still square to the rule, and true to the plummet; and every mystic line within it literally squares a circle!

THE BOSS AND THE DIAGRAM.

Let us now examine the boss upon the granite leaf.

To the boss upon the granite leaf we have already frequently alluded. It is a small raised ornament upon its upper block. It is not located at the centre of this block, but seems to be purposely displaced therefrom a short distance to the west and downward. This displacement amounts to exactly one inch in westing. It is, in fact, equal to one of the grand earth-commensuric standard units of the polar axis.

As though to intensify this reference to the unit inch, the height, or relief, of the boss is also an exact inch. Plaster-of-Paris casts of this feature of the ante-chamber have frequently been made, and taken to England, where they have been studied with some remarkable results.

The following are some of the more noticeable discoveries that have thus far been made with reference to it.

Its cubical contents, or volume, is one pint; and, as this pint weighs an even pound when filled with water, we are forced to regard the boss as an epitome of unit measures. The inch, the pint, the pound, — the three working-units of length, capacity, and weight, — are thus united in this wonderful little projection.

Now, the shape of the boss is equally remarkable. Its base is a circular arc of rather more compass than a semicircle. It is limited at its lower edge by a horizontal chord which is exactly 5 inches long, or a span; i.e., one-fifth of a cubit. Nor is this all.

The radius of the circle thus limited by the chord of 5 inches is 2.820 + inches. Mathematically this is, perhaps, the most wonderful dimension of the boss; for a circle whose radius is 2.820 + inches, has the same area as a square whose side is 5 inches! Thus the boss is at once both a practical example and symbol of the circle squared.

No more beautiful geometric representation of this celebrated problem could be conceived of, than to cut a circle by that particular chord which is the side of its equal square. This symbol appears to have been, not only purely original with the architect of the Great Pyramid, but to have been monopolized by the boss from that day to ours, — the one of its discovery.

Let us now apply the pyramid diagram to this boss.

Let its projecting base of 5 inches, or a span, be the base of our model pyramid and of its vertical triangular section: with the radius of the boss describe the equal area circle, and it is manifest that the whole diagram thence results in its most absolute simplicity, and that it involves the height, length of circumference, radius, chord, and diameter, or breadth of the boss, in their actual mathematical positions. The boss, therefore, may be as beautifully renewed by this mysterious diagram as we have already shown that the ante-chamber may be.

While studying the position of the boss, Mr. St. John Vincent Day, an English civil engineer, discovered that the distance from its centre to the end of the leaf itself, in its well-cut groove of the eastern wainscot, was exactly one sacred cubit, or 25 inches. Thus, as plainly as a stone can speak, this granite leaf tells us that 5 inches make a span, and 5 spans make a cubit.[1]

In 1875 another practical engineer, Mr. Waynman Dixon, writing to Professor Piazzi Smyth concerning the granite leaf, makes use of the following language: "The more I see of this remarkable stone, or leaf, the more I am convinced that the

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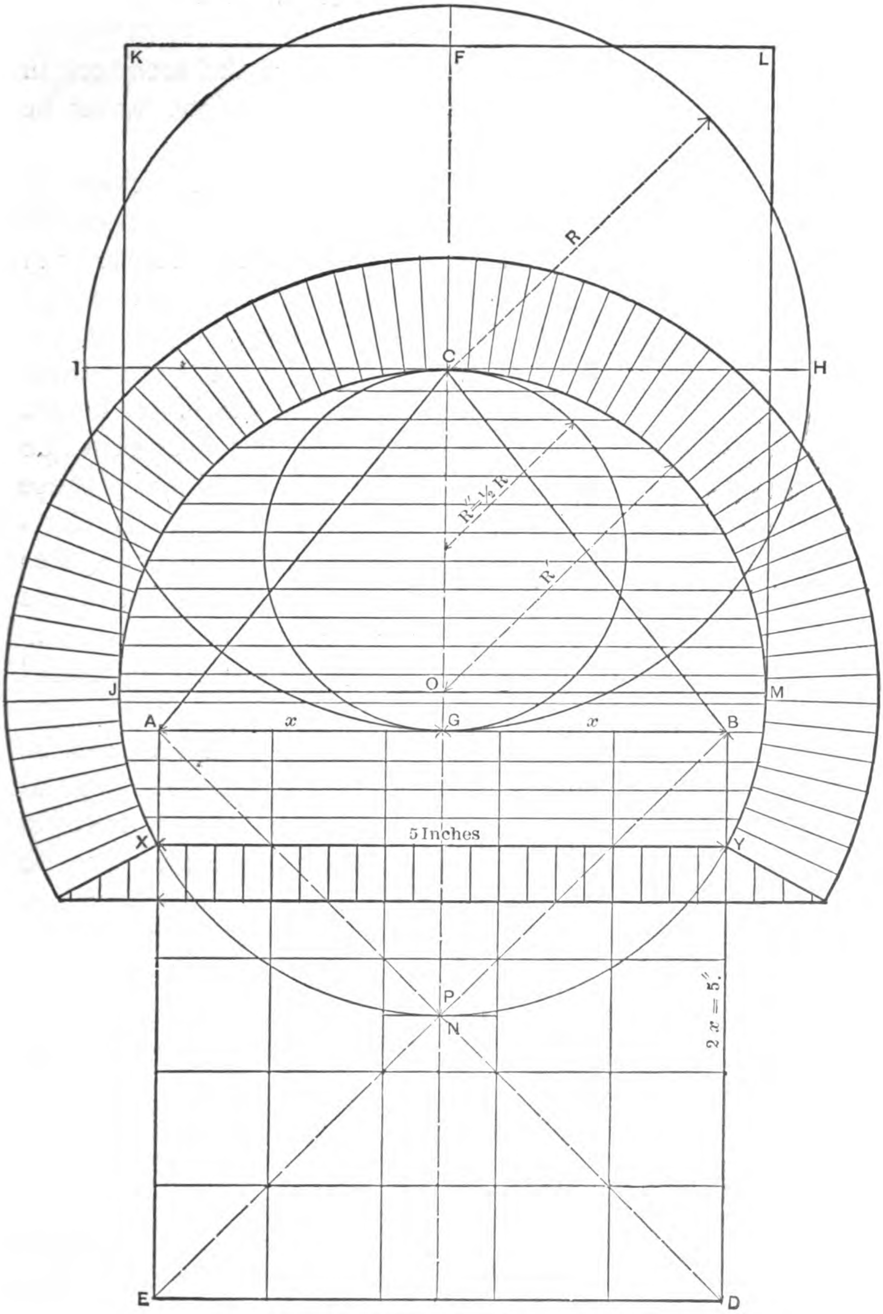
[1] See Fig. 15.



THE PYRAMID DIAGRAM AND "THE BOSS."

(ITS IDEAL SHAPE RESTORED.)

Fig. 16.

upper irregular part *is in its original condition*, and not broken away by specimen-mongers or Arabs."

Now, what could have been the *intention* of the architect in leaving this upper surface so irregular that its height cannot be *accurately* measured?

This question was answered by the Rev. C. W. Hickson in 1877, with a rigid mathematical demonstration. The length and width of the leaf can be accurately obtained, because they are clear and square cut. They are respectively 41.2 and 15.7 inches. The only way, however, that we can arrive at the height of the leaf, is by taking a mean: this is the true method, by modern rules, of mensuration. Now, the maximum height is 51.3 +, and the minimum height is 45.8 +, the mean of which two is 48.55 +, possibly inclining to 48.57. Multiplying these three dimensions together, to obtain the volume of the leaf, its cubical contents comes out in the wonderful sequence of figures known as the circummetric ratio. It is 31415.9 +, etc., cubic inches, or 10,000 times π cubic inches. This is a most stupendous repetition of the π references that we have already found to exist in the shape and dimensions of the boss itself.

The contents of the lower block of the granite leaf can be most accurately determined, because of its perfectly rectangular finish. This is found to be an even fourth of the contents of the coffer; or since the latter is a "caldron," or four English "quarters," the contents of the lower block of the leaf is one such English, Anglo-Saxon, or, rather, pyramid, "quarter."

Who can fail to be astonished at such remarkable discoveries? And who can doubt the importance of a system of metrology, for the preservation of which such infinite pains seem to have been taken.

But we are by no means through with the boss upon the granite leaf. I shall now ask you to follow me in a rapid glance over some other of my own discoveries in this neighborhood.

It will probably have been noticed, that, in all the representations you have seen of the "pyramid diagram," there has been one line to which, as yet, no reference has been made. It is the line *XY*. This is the right line joining the two points in which the circle of equal area to the pyramid base intersects the square which represents that base. Its length is, of course, equal to the side of the square, or to the base of the pyramid triangle.

Now, this line is one of the most important ones in the entire diagram; though I have not the slightest doubt but that its importance, and its relation to the diagram, have entirely escaped the notice of every one present. It certainly escaped my own for many months. So completely did the importance of this line escape me, that I do not believe I was even led to construct it until, in my study of the diagram, I had followed its applications down throughout the pyramid to the very boss upon its granite leaf, and found in it its actual ideal. When I finally came to apply this diagram to the boss, I was *forced* to draw this line, because it was one of its most expressive features; namely, its *chord*. In such a diagram, complicated by its very nature, the utmost simplicity consistent with the ends in view was, of course, requisite; and, had it not been for the necessary introduction of this new line XY when we arrived at the boss, it would probably have been undrawn to this day.

But even such a forced construction did not at once point out its supreme importance. Its discovery was entirely due to an accident, but to a most natural one, and to one that is beautifully in keeping with those that have led to the discovery of almost every secret that has been wrenched from the Great Pyramid.

THE WORKMAN'S RULE FOR SQUARING THE CIRCLE.

I had no sooner completed the figure showing the application of the "pyramid diagram" to the boss, than the fact of the five-inch length of its upper chord suggested to me the idea of dividing the chord off into its five several parts or inches. This I did, and continued the vertical lines of division downward across the square. It then most naturally occurred to me, as a sequence, to redivide it by a set of horizontal lines, so as to represent each of the 25 square inches which enter into the square of the chord itself.

While constructing this 25-part (or cubit) division, I noticed with surprise, that, so close as I could draw the figure at its own actual scale, the first horizontal division *fell coincidently with the chord of the boss itself.* It at once struck me that this fact afforded a practical method of *squaring the circle.* I shall henceforth denominate this method the "Workman's Pyramid Rule."

As shown in the "Workman's Diagram," you will notice that the circle of equal area to the pyramid base, passes not only through the vertex of the pyramid triangle, but also through the centre of the square base. Now, the new discovery revealed the additional fact, that it also *practically* passed through two other points, *X* and *Y*, which lay an inch, or one-fifth, downward upon each of the vertical sides of this square.

Now, but one circle can be passed through three given points, *X*, *Y*, and *N*; and since in the diagram this circle is the equal-area circle, it follows, that to the same degree, in every square, an equal-area circle may be passed through its centre and two other points situated one-fifth of the way down two of its opposite sides.

Any line can be divided into five equal parts by a most simple geometric construction. No one can study the boss upon the granite leaf without having its one-fifth (or, in *its* case, "an *inch*") division sooner or later suggested to his mind. The "Workman's Diagram" shows that this fraction (one-fifth) is not only the lowest term of a most useful approximation, but one whose error cannot be appreciated by the ordinary workman.

But this error, slight as it is without *any* correction, can be made to vanish in a most simple and practical manner, to a point that is far beyond the power of recovery by means of the best micrometer yet constructed.

This is of enough importance to explain at greater length.

The radius of the boss, calculated by the best modern π value, so that its circle shall have the same area as the square of its five-inch chord, comes out 2.820947+, etc., inches.

If, however, this radius be geometrically *constructed* by means of the "pyramid rule" just given, it will measure 2.833333, etc., inches, *ad infinitum.* The difference between these two values is but .012(385) inches.

This difference is actually less than the width of the mathematical line that an architect would have to make in drawing a working-plan of the boss, let him execute it with the utmost care.

Until mathematical instruments have been so perfected that distinct working diagrams can be drawn in lines twice as fine as jet-black *hair*, we need feel no concern at errors of construction so minute.

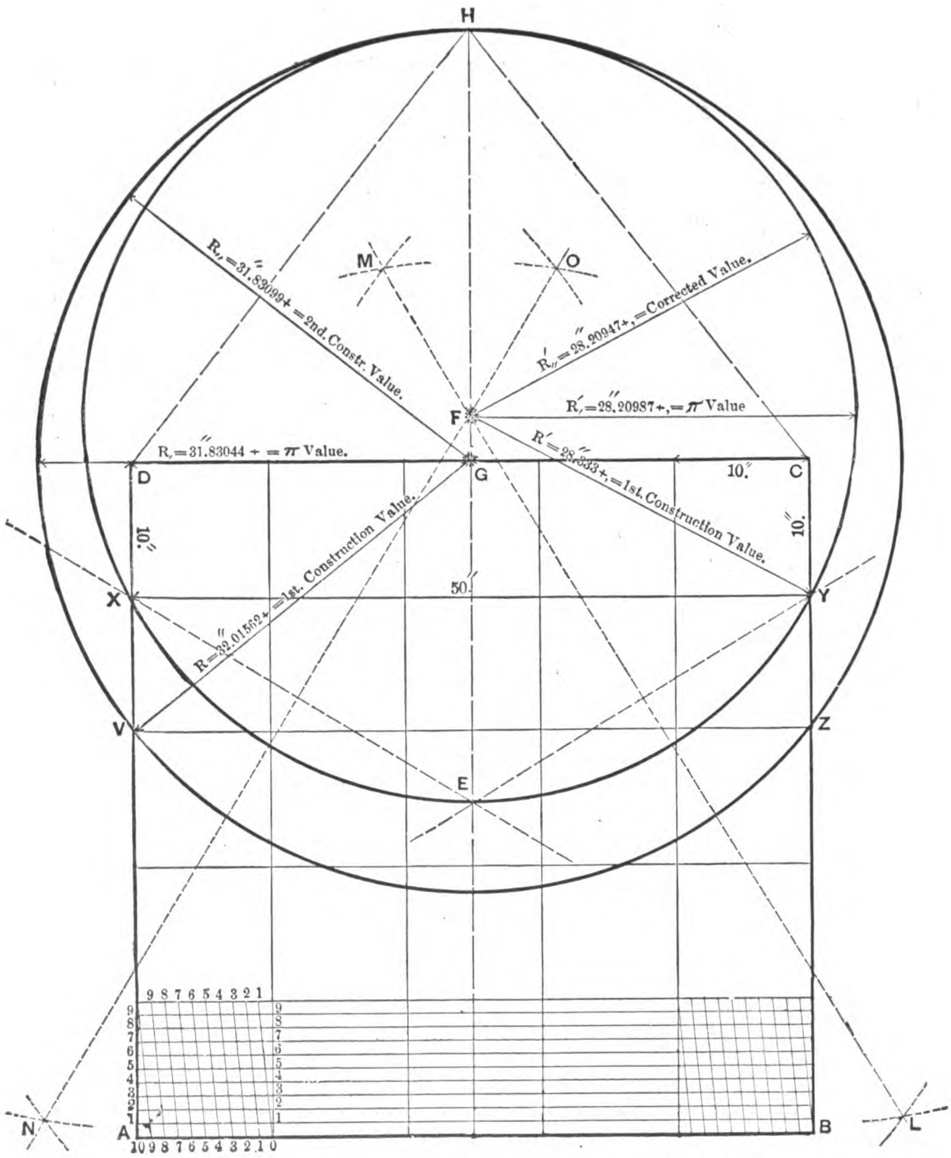
The standard mathematical breadth of a human hair is one forty-eighth of an inch. This is about the width of the least practicable line of construction that could be advantageously employed in making a suitable diagram of the boss at its own actual scale. And yet such a line — one that had only a hair's-breadth — would completely cover all the error introduced by the proposed construction. Its inner edge would fall about as much within the true circle as its outer edge (drawn coincident with the line of construction) would fall beyond it. The centre of a hair-line would, in fact, just about indicate the actual position of the true circle.

The absolute width of the belt of error that surrounds this construction is about $\frac{1}{81}$ of a unit (the inch in this case); i.e., it is $1 \div (9)^2$. If, therefore, the circumference, to be constructed by the "pyramid rule," be made $\frac{1}{81}$ of the unit in width, its inner edge, just at the vanishing-point, will indicate the absolute position of the required line to a degree of accuracy whose error the microscope could hardly grasp ($\frac{1''}{25000}$).

To illustrate better the practical application of this *workman's* "pyramid rule" for squaring the circle, I shall take the largest diagram that I can conveniently handle. The cloth before you is 90 inches square, and is short by about 30 inches of what I need to complete the whole pyramid diagram. (Fig. 17 is a reproduction at $\frac{1}{36}$ of the diagram that accompanied this lecture.)

Let *AB* represent the side of a square 10 times as long as the chord of the boss. It is 50 inches in length, or two cubits. This square contains 2,500 square inches. We wish to obtain an equivalent circle; that is, one whose area shall contain 2,500 circular inches.

By the pyramid rule, divide the side *CD* into 5 equal or "unit" parts; lay off *DX* and *CY* each equal to one such "unit;" through *XE* and *Y* pass as fine a circumferential line as possible, and consider it the outer edge of the bordering. Make the width of the mathematical line that is to represent



THE WORKMAN'S RULE FOR "SQUARING THE CIRCLE."

Fig. 17.

this circumference, or border, equal to unity, $\div 9^2$; i.e., equal to Cc', or, in this case, equal to $10'' \div 81 = .12345''$, etc.: then will the inner edge of the border, thus constructed, limit the circular area required, and that area be correct to within less than .07″, a square inch, or equal to $\frac{1}{36000}$ of this desired area. One could buy gold and precious stones by such a rule without fear of appreciable loss.

The radius R', passing the points XY and E, and limiting the outer edge of the border, is 28.33, etc.; the radius $R'_{\prime\prime}$, corrected by $\frac{1}{(9)^2}$, and limiting inner edge of the border, is 28.209877; thus $R'_{\prime}$, depending upon π values, is 28.209479.[1]

But, furthermore, since CDH is manifestly a pyramid triangle, all of the properties and relations of the pyramid diagram result from one construction. Thus,

$$\pi \overline{FY}^2 = \overline{DC}^2$$

as just demonstrated;

$$2\pi HG = \text{perimeter } ABCD = 4CD$$

$$\pi \overline{HG}^2 = (2FY)^2$$

and

$$HDC = \overline{FY}^2 = \pi(\tfrac{1}{2}GH)^2, \text{ etc.}$$

This is a practical method of constructing equivalent squares and circles, either as to area or perimeters, the equal of which has hitherto been unknown to modern mathematicians, and the discovery of which certainly enhances the interest centred in the boss upon the granite leaf or "standard bar."

But, before finally turning from this subject, I wish to call attention to another phase of the method I have given. Its discovery was even *more* purely accidental than that by which we have been able to locate the position of the equal-area circle. It depends upon the same method, and is most beautifully in accord therewith. Before completing the diagram before you, it seemed desirable to place upon it, though merely for purposes of comparison, *the circle of equal perimeter* to the given square. The cioth, being too limited to place this circle in its

[1] In Fig. 17, $R'_{\prime\prime}$ should $= 28.20987$, and $R'_{\prime} = 28.20947$, the values having become misplaced on the engraving.

proper place, as shown in the regular pyramid diagrams, I was forced to draw it (with, of course, the height of the pyramid triangle as a radius, but) with the centre of its base as its centre instead of the vertex of the triangle, as in the most natural method. Upon describing this circle, I was again surprised to see that *it* also passed through two noticeable and corresponding points upon the sides of the given square. These points, you will perceive, are situated as far below those which mark the intersection of the equal-area circle with the given square, as the latter are below the angles of the square.

In other words, the circle of equal circumference practically passes through two points situated one-fifth or a unit distance below those which locate the circle of equal area.

Thus the same principle holds good. Comparison with the true π value shows that the error in this latter construction is also of no moment, but that, when desirable, it may be made to vanish out of microscopic sight by applying the several indicated corrections.

Thus, by simply deducting the primary correction of $\frac{1}{54}$ $= \frac{1}{2.(3)^3}$ of unity, the radius becomes 31.83044 +, which is only .00054″ too small. This radius gives a value for the circumference of 199.9965″, or one that is short of the true one (200″) by but .0035″, or .00035 of unity; or, finally, one that is short of the whole circumference sought by but $\frac{1}{57111}$, or about $\frac{1}{60000}$ thereof. The simplicity of this graphic method of practically squaring the circle, its entire originality with the Great Pyramid, and its direct and logical deduction from its rediscovered and most important diagram, are certainly matters of deep interest to all students now interested in pyramid metrology.

UNITY REFERENCES OF THE BOSS.

Before turning from our special mathematical consideration of the boss upon the standard bar, I must call your attention to a most remarkable addition to the unity references already noted as centring in and around it. We have found in it the inch, the pint, and the pound: we have also seen how beautifully it satisfies what has been called "the pyramid method."

In the present lecture we have likewise seen in it an ideal realization of the "pyramid diagram," and such a suggestive one as to afford us a practical solution — a workman's solution — of the problem of problems.

If the pyramid itself be a common multiple of earth-commensuric references, then especially so is its safely guarded boss. From every stand-point it is wonderful.

It was while studying the mutual relations of its inch height, its five-inch chord, and its special radius, that I noticed its indicated circumference was none the less worthy of attention, and that it could be factored into a most astonishing series of circular unit-references. Thus,

$$
\begin{aligned}
2\pi R &= 17.7245+,\text{ etc.},\\
&= 2.\ (3.14159,\text{ etc.})\,(2.82094,\text{ etc.}),\\
&= 2.\ (3.141,\text{ etc.})(.5641)\,(5)\,(1 = H),\\
&= (6.283,\text{ etc.})\,(.5641)\,(H = 1)\,(.15915)(3.1415,\text{ etc.})\,10,\\
&= 2\ (6.283,\text{ etc.})\,(.5641)\,(H = 1)\,(.15915)\,(3.1415,\text{ etc.})5.
\end{aligned}
$$

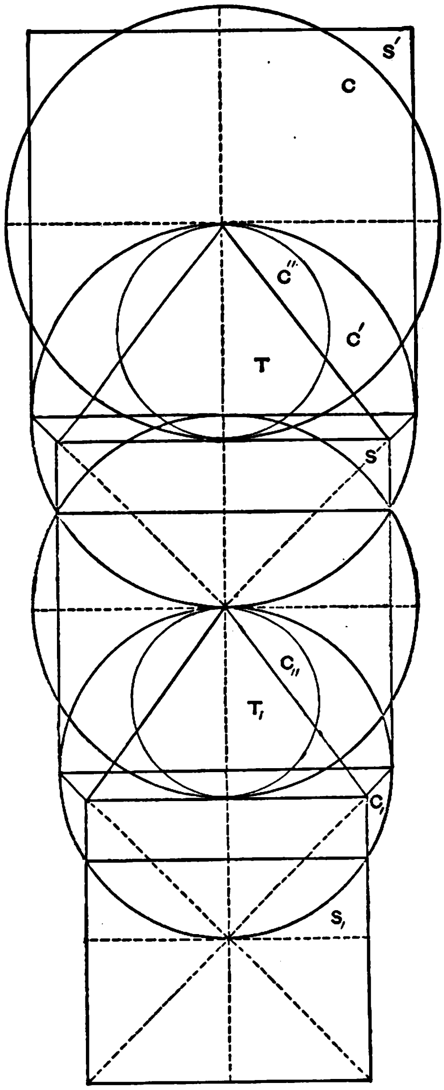
Or, translating these apparently unmeaning figures into words, we have, as follows: —

$2\pi R = 17.7245+$, etc. = the circumference of the boss = 2 (circ.$^{\text{rad.}-1}$) (rad.$^{\text{area}-1}$) (boss$^{\text{height}-1}$) (rad.$^{\text{circ.}-1}$) (area$^{\text{rad.}-1}$) chord of the boss.

That is, twice the product of five factors that are intimately related to unity, by five units or the chord of the boss itself, is equal to the circumference of the boss.

GENERAL FORMULAS AND THE PYRAMID DIAGRAM.

The same set of relations which I have lately pointed out in the "International Standard" for September, 1883, as existing between the lines and dimensions of the "Latimer diagram," as I there completed it by the addition of the circle whose diameter is 81, are found to run through the pyramid diagram as here given. Thus, in the double diagram, let $S,$ be first square, $C,$ the first circle, S the second square, C' the second circle, S' the third square, and C the third circle. Then these relations are as follows: —



THE DOUBLE DIAGRAM.

(KING'S CHAMBER, ETC.)

Fig. 18.

I. The diameter of the first circle is the side of the second square, that of the second circle the side of the third square, and generally the diameter of any circle (say, the nth) is equal to the side of the next (i.e., the $n +$ first) square.

II. The perimeter of the first square equals the circumference of the second circle, that of the second square equals the circumference of the third circle, and so on; so that, generally, the perimeter of the nth square equals the circumference of the $n +$ first circle.

III. The area of the first square equals that of the first circle, that of the nth square equals that of the nth circle.

IV. A pyramid triangle may be based upon the diameter of any circle, as, for instance, that of the first, and have for its height the radius of the second therefrom. Thus, the diameter of the first circle ($C_{\prime}$) and the radius of the third circle (C) give us a pyramid triangle, and the diameter of the nth, and the radius of the $n +$ second, another.

From the foregoing rules, it follows, that since to the diameter of the first circle, there can be given any numerical value whatever from 0 to infinity, such a sequence of squares and circles may be made to represent the nucleus of any desirable series of pyramid triangles; and hence, that there is a series which corresponds to the base side of the Great Pyramid as the side of a second square and the diameter of a first circle.

Now, it is a fact that the height of this pyramid, so far as we can now measure and estimate, be it in the terms of any linear measure whatsoever, corresponds to the radius of a third circle of such a series, while its base corresponds to the side of the connected second square: hence we are justified in believing that the Great Pyramid of Gizeh was built as an intentional exponent of such a geometrical series, rather than that it is a gigantic accident. To this conclusion we are forced without reference to any knowledge of the absolute unit of measure actually employed by its builders.

From a study, however, of these relations, as recast into what we have here termed, *par excellence*, the "pyramid diagram," and from a further study of the wonderful mathematical connections based upon the 25-inch, or sacred, cubit, we are forced to the conclusion that this particular measure, based upon the earth-commensuric unit inch, and so extremely close to our own Anglo-Saxon unit, is the true metron of its builders.

Before leaving this subject, therefore, let us so generalize the relations we have discovered, as to determine, in the terms of some common quantity, the whole series of inter-related values.

Denote the square base of the pyramid by S, the pyramid triangle by T, the equal-perimeter circle by C, the square whose area is equivalent to this latter by S', the equal-area circle by C', and the circle on the altitude of T by C'', and designate, by corresponding notation, the elementary parts thereof. Let, also, $C_{,}$ represent the circle upon the side of S as a diameter, $C_{,,}$ the circle upon R' as a diameter, $T_{,}$ the pyramid triangle of equal area to $C_{,,,}$ and $S_{,}$ the square whose area is equal to that of $C_{,}$, etc.

These circles, squares, and triangles form the sequence known as the *double* pyramid diagram. Commencing at S, they run upward through the links of the upper half of the diagram so far as C''. Commencing, also, at the same point S, they run downward through those of the lower half of the same diagram. The square S and its equivalent circle C' are common to each half of the double diagram.

The preservation of this system of notation will greatly facilitate the ready designation of any particular square, circle, or triangle in this confusing series. We shall now enumerate the formulas governing the functions of these several mutually related figures, commencing with those of S.

S. Denote the side of S by $2x$. Then

$$S = 2x;\ \text{area } S = 4x^2;\ \text{perim. } S = 8x;\ \text{diag. } S = x.2\sqrt{2}.$$

$$T.\quad \text{Base} = 2x;\ \text{alt.} = \frac{4x}{\pi};\ \text{perim.} = 2x\left(1+\frac{1}{\pi}\sqrt{\pi^2+16}\right);\ \text{area} = \frac{4x^2}{\pi}.$$

$$C.\quad R = \frac{4x}{\pi};\ D = \frac{8x}{\pi};\ 2\pi R = 8x;\ \pi R^2 = \frac{16x^2}{\pi}.$$

$$S'.\quad s' = \frac{4x}{\sqrt{\pi}};\ P' = \frac{16x}{\sqrt{\pi}};\ \text{area}' = \frac{16x^2}{\pi};\ \text{diag.}' = \frac{4x\sqrt{2}}{\sqrt{\pi}}.$$

$$C'.\quad R' = \frac{2x}{\sqrt{\pi}};\ D' = \frac{4x}{\sqrt{\pi}};\ 2\pi R' = 4x\sqrt{\pi};\ \pi(R')^2 = 4x^2.$$

$C''.\quad R'' = \frac{2x}{\pi};\ D'' = \frac{4x}{\pi};\ 2\pi R'' = 4x;\ \pi(R'')^2 = 4x^2.$

$C_{,}.\quad R_{,} = x;\ \ D_{,} = 2x;\ 2\pi R_{,} = 2\pi x;\ \pi(R_{,})^2 = \pi x^2.$

$C_{,,}.\quad R_{,,} = \frac{x}{\sqrt{\pi}};\ D_{,,} = \frac{2x}{\sqrt{\pi}};\ 2\pi R_{,,} = 2x\sqrt{\pi};\ \pi(R_{,,})^2 = x^2$

$T_{,}.\quad \text{Base} = x\sqrt{\pi};\ \text{alt.} = \frac{2x}{\sqrt{\pi}};\ P' = x\left(\sqrt{\pi} + 2\sqrt{\frac{\pi^2+4}{\pi}}\right);\ \text{area} = x^2.$

$S_{,}.\quad S_{,} = x\sqrt{\pi};\ P_{,} = 4x\sqrt{\pi};\ \text{area}_{,} = \pi x^2;\ \text{diag.}_{,} = x\sqrt{2\pi}.$

An application of the foregoing formulas to any value of x will result in the determination of the corresponding numerical values of the entire series resulting therefrom.

Thus, if such a series of values be constructed upon the earth-commensuric unit inch as the side of $S_{,}$, there will result what may be properly termed the "unit diagram." Now, if we redraw the diagram at a scale five times greater, the "span or boss diagram" will result. And, if this latter series of values be again multiplied by five, there will result the "cubit diagram."

The "pyramid diagram" itself is determined from the "cubit diagram" by multiplying all of its lines by the year number 365.242±. This latter diagram, multiplied by 10,000,000, gives us, so to speak, the "terrestrial diagram;" and undoubtedly, as the series runs upward, we may pass through "solar" and "sidereal" diagrams in our progress towards the ultimate and "universal diagram," whose terms of infinite scope span all the borders of creation. Descending again, however, from the numerical values of the "pyramid diagram," we find, that, taking one twenty-fifth thereof (the cubit number), we are landed at a diagram which locates the dimensions of the king's chamber; and still further, that one-fourth thereof, or one-hundredth of the "pyramid diagram," yields us that of the ante-chamber. In a similar manner, one-fifth of the king's chamber gives the "coffer diagram," etc. It seems inevitable that the queen's chamber, the subterranean chamber, and the yet to be discovered "royal arch-chamber" of the monument, will yield similar diagrams, with multipliers (perhaps 52.17+, the year of weeks; 7 cubits = 175″, π values, etc.) just as expressive, and of just as important value to metrology.

From another stand-point, since 116.26 is the ratio of $\pi : Y = 3.14159+$, etc. : $365.242\pm$, etc., and since this is the governing dimension of the ante-chamber, this latter may have its diagram founded directly upon π values themselves. Thus, the "pyramid diagram" in cubits, divided by π, gives us the "ante-chamber diagram," in inches, etc. The very passages individually, and as a scheme collectively, will undoubtedly be likewise found to yield themselves to such a diagram: indeed, I have already determined some of them; though space forbids their enumeration here.

THE CUBIT, THE TWENTY-FOUR INCH GAUGE, AND THE YARD.

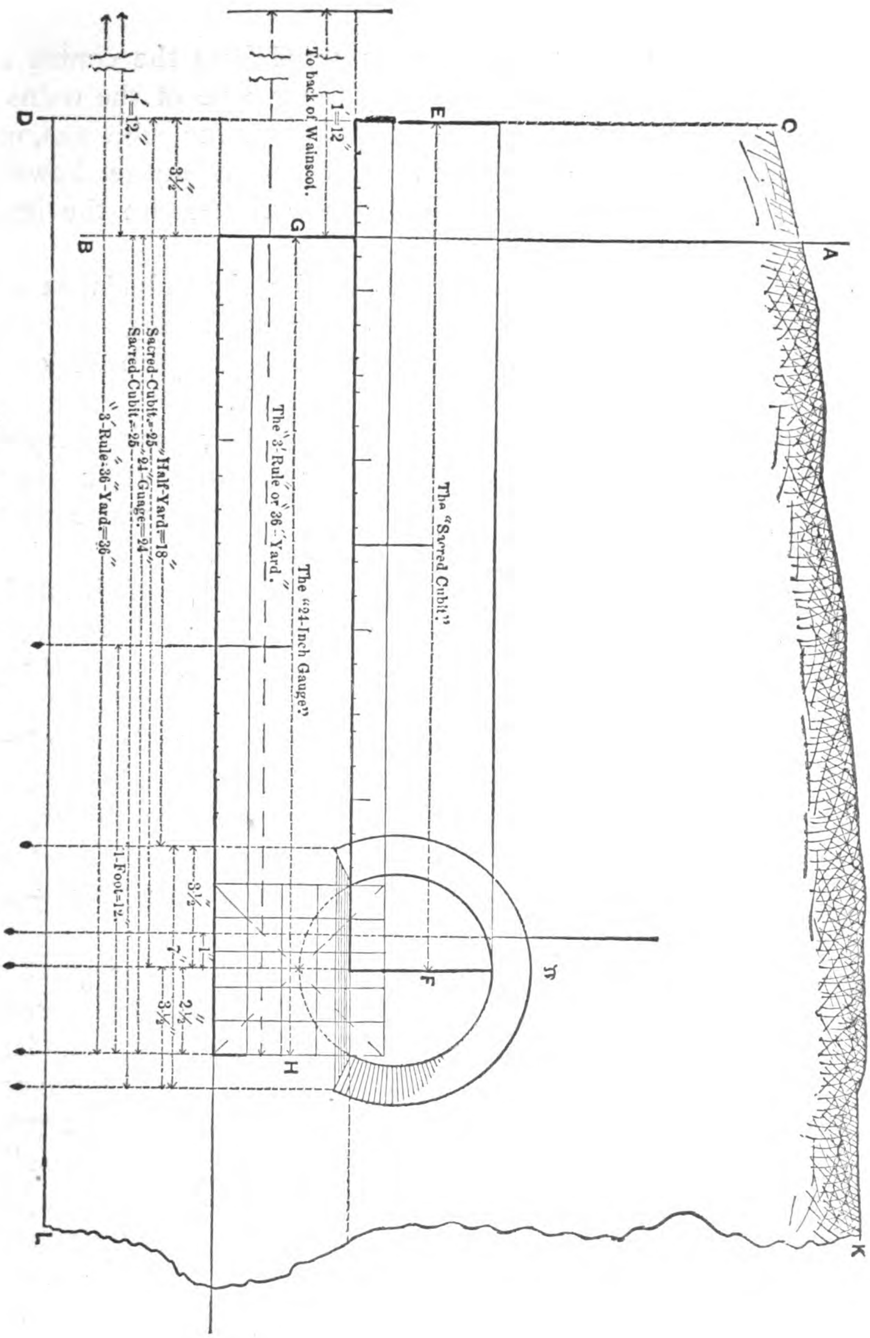
The sacred cubit of the Jews was set aside for special purposes. With it they proportioned the tabernacle and the temple. It had peculiar and important uses in the mystic rites of the sanctuary.

Whether, however, this *sacred* cubit was ever put into the hands of the common workman has long been a subject of controversy. It is probable that the laborer, at his ordinary task, did not use this sacred length: indeed, we know that the Israelites employed, in their every-day affairs, a cubit of far different length. This is sometimes spoken of as "the cubit of a man;" it was a span shorter than the sacred cubit, and was closely assimilated, if not exactly equal, to the cubits of Egypt and the surrounding nations.

There were, in fact, several such every-day cubits in use among the Jews; and, included in this number, was one in particular, that is by far the most important. This cubit is the origin of the modern "foot," which is inherited by almost every northern European nation from their ancestors. It was different from every Egyptian cubit, and is still the true workman's measure.

It is best known in our day as the two-foot rule, or the celebrated "24-inch gauge" that has been so carefully handed down from pyramid and temple times by the Freemasons, and is now to-day in the hands of every operative mason in the world.

It is of that special length which is still found to be most convenient by masons and carpenters, and it is as beautifully



THE "STANDARD BAR," OR "GRANITE LEAF."

(ITS LINEAR SIGNIFICANCE.)

Fig. 19.

indicated upon the granite leaf of the Great Pyramid as is the sacred cubit itself.

In obtaining the latter, we measured from the centre of the mystic boss, and penetrated into the groove of the wainscot to the very eastern end of the leaf. The sacred cubit was, in fact, hidden from ordinary sight. The 24-inch guage, however, is exposed to the full view of all who will measure the leaf, and live by its masonic teachings.

No sooner was I convinced of the rightful claims of this granite leaf to another and more expressive designation, that of the "standard bar," than I went deliberately to work to find upon it this 24-inch, or workman's, gauge.

The very simplest investigation of the dimensions exposed to view, showed me, that, from the western extremity of the chord of the boss to the face of the eastern wainscot, was exactly two feet, or the length of the masonic standard.

This workman's gauge is thus most *conveniently* located upon the standard bar for measure, for scientific subdivision, and for comparison with the wonderful sacred cubit that rests above, and upon it.

But another Anglo-Saxon measure of note is now brought prominently to our view: it is the yard itself. The thickness of the eastern wainscot is just 12 inches, or the "common foot:" hence from the wall through the wainscot, and to the western extremity of the lower chord of the boss, is $1' + 2' = 3' = 36'' =$ the Anglo-Saxon yard.

From its remarkable commensurability with the great *magistral* line of the whole earth, — the polar axis, — the sacred cubit lays direct claims to precedence over all other standards of linear measure. We have already alluded to this quite particularly, and have, throughout our investigations of the pyramid, seen how clearly the sacred cubit and its "unit inch" were ruling elements in the combinations that conceal its mysteries. The earth-commensuric "inch" must be the integral part of every standard of linear measure raised against the monument of Egypt, if we wish to discover there aught besides its symmetry. To the "metre" it is silent, and to the centimetre a hundred times as dumb.

By means, however, of the earth-commensurability of the inch and sacred cubit, and of their axial reference, we can pass understandingly beyond our world, and measure the radius of its orbit, and the various other distances of the solar system, in terms of the polar radius. Indeed, mounting a step higher, the astronomer estimates the distance from *our* system, — or its centre, the sun, — to the fixed stars, and uses for this purpose the radius of the earth's orbit.

This most natural growth of standard out of standard may go on forever throughout the vast and illimitable regions of space, without ever losing its earth reference. The axis, the cubit, the inch, accompany us in each step outwards, go we how far so ever; and all our information can be harmonized into a vast system of linear measure, that, commencing among the little inches of ordinary life, at last span out the whole creation with scale indeed stupendous, but never quite beyond our comprehension.

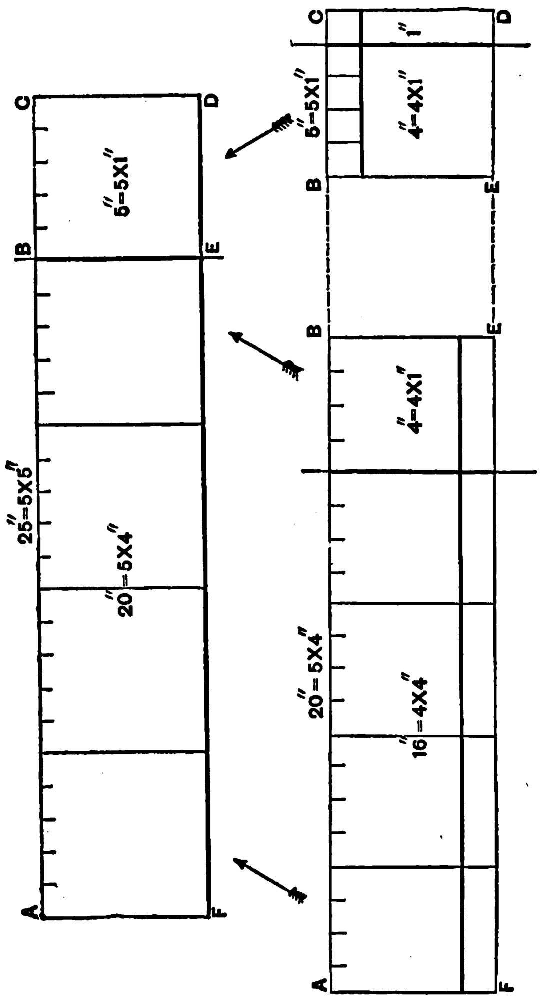
From what we have seen to-night of the pyramid diagram, and its perpetual harmonious repetition throughout the building, who, then, can doubt but that the more gigantic harmonies of the universe are just as beautifully balanced, and that after we have expressed the harmonies of our *own solar system* into a pertect geometric figure, or solar diagram, so to speak, we may thereafter, by the careful measure of the greater radii beyond us, — magistral lines of other, but still similar, diagrams, — learn also *all* about the universe that lies within the reach of our invention and discovery?

But, over and above its *earth*-commensuric claims, the sacred cubit possesses other properties that mark it out as unique among all the standards of linear measure known to man.

As our time is drawing to a close, I can but barely enumerate a few of the more wonderful facts concerning *the cubit as a standard*, and which I have been fortunate enough to discover in my pyramid studies.

THE FIVE-PART DIVISION OF THE CUBIT.

From its susceptibility of a five-part division (i.e., into five spans), the cubit takes to itself all of the circular properties we have just noticed in the boss. A "pyramid diagram," constructed upon a chord that is five *spans*, instead of five *inches*,



THE FIVE-PART DIVISION OF "THE SACRED CUBIT."

Fig. 20.

in length, will, of course, possess equivalent properties, and show forth the same absolute ratios. This is the first point of peculiar interest to which I ask your attention. I shall recur to it more particularly later on.

Let us now separate one of these end spans from the other four by a *marked* line of division. The cubit is thus divided into two very important parts; the one containing twenty unit inches, and the other five of them.

Now, it is one of the noticeable features of this division, that each such part contains a whole number of inches, to wit, five and twenty; and, furthermore, that each part may, in turn, be similarly *re*subdivided into five smaller parts, and that this redivision may still be effected without the rupture of the unit.

By this process the "span," or smaller part of the cubit, is reduced to its ultimate unit, — the inch.

In this respect the sacred cubit differs essentially from the foot, the 24-inch gauge, the yard, etc. None of these standards can be subdivided into fifths without destroying the integrity of some of their earth-commensuric units; and, unless the standard be divided into fifths, it cannot be employed as the basis of circummetric scales. Take, for instance, the yard: like the cubit, it numbers a perfect square — $36'' = 6 \times 6''$. Now, if the side of a square be 6 inches long, its area will be 36 inches, and its perimeter 4×6, or 24 inches. But $24 + 6 = 30''$; so that there remains an unused surplus of six inches, by which the yard is dissimilar in construction to the cubit.

The French metre must be entirely prohibited from entering into such a comparison. As we have already shown, it is not truly earth-commensuric, is erroneous in principle and calculation, and certainly is not a *scientific* standard for international intercourse. Moreover, it is, in reality, *the unit* itself of the metric system; and the method does not propose to *divide* a *unit*, but simply to *obtain* the latter by the subdivision, into fifths, of the standard.

The metre is decimal in its construction: so, too, may any unit be. Of course the division by fifths may proceed through any decimal system: but, if we admit into the comparison a standard whose essence is this decimal system, then all standards whatsoever are admitted through the same door; for they can all be made thus decimal.

Moreover, to illustrate this important characteristic of the cubit even more plainly, suppose it also to be decimally divided into 100 equal parts. A fifth of these parts, or $\frac{20}{100}$ of the whole cubit, is still an even number of *inches;* i.e., 5 inches: and one-fifth of this, or of $\frac{20}{100}$, is also an even number of inches, or the unit itself.

With the metre, however, it is different, even in its decimal relation. Its length is 100 centimetres. This is a perfect square: but its fifth is 20 centimetres, the square of which (400) does not give the value of the metre; though the perimeter of this square (80) equals the cut-off portion.

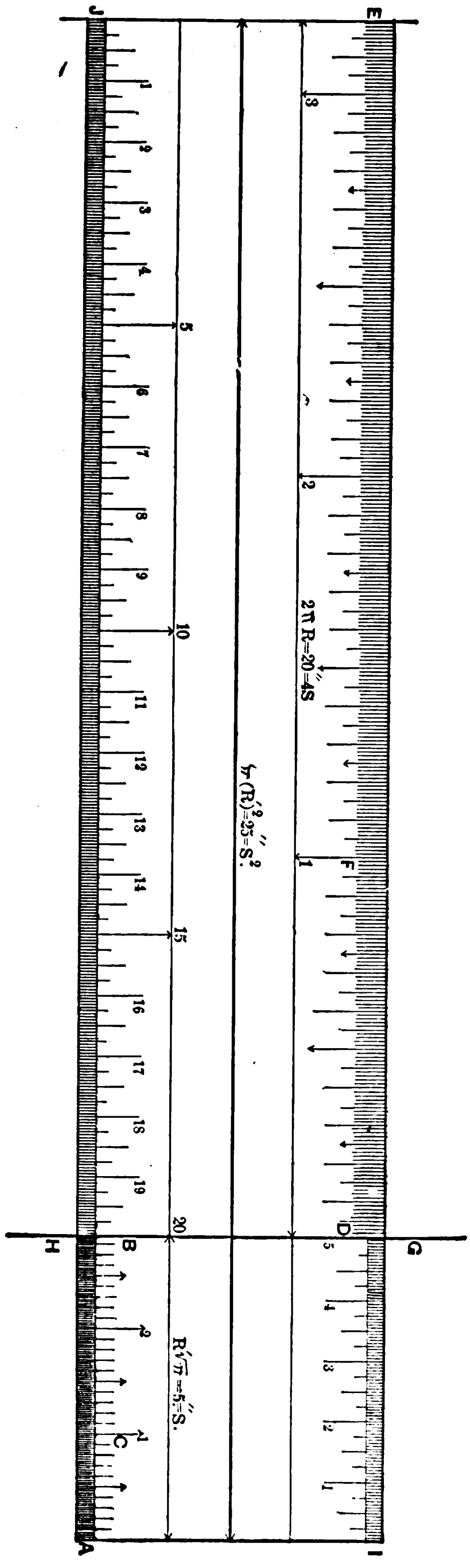
The cubit is, in fact, as purely decimal as any standard can be, but with the overwhelming advantage over all others, that every simple fractional part thereof, whether decimal or common, is truly earth-commensuric. It has not only a direct ratio to the axial radius of the earth, but through it to the universe beyond.

At the first glance it seems as though the 45-inch ell of cloth measure likewise possesses this 5-part subdivision without the rupture of the unit. The ell is a yard and a quarter, or five-quarters of a yard, long, and is thus by construction divisible into 5 equal parts. Each of these parts is 9 inches long. But the process so well begun stops here. The "quarter" is not the unit of the ell: its unit is a "nail," $2\frac{1}{4}$ inches in length. Now, one-fifth of 9 inches is not $2\frac{1}{4}$ inches, but $1\frac{4}{5}$ inches; and thus the unit inch is ruptured fruitlessly. Moreover, had the unit "nail" itself resulted, it would have been only a *broken* number of inches, and not suitable for earth-commensuric purposes. The rule, in fact, universally fails except in the 25-inch cubit. Thus the cubit is unique in its five-part division, repeated until its own unit — the earth-commensuric inch — results.

THE CUBIT, THE SQUARE, AND THE CIRCLE.

Let us now see into what some of these properties develop when thus united in a single standard of linear measure.

From the stand-point to which we have attained, it should be clear without any further investigation whatever, that, as the cubit has been shown to be so important a line in the pyramid



THE CUBIT, THE SQUARE, AND THE CIRCLE.

Fig. 27.

diagram, it *must* possess some remarkable powers for measuring squares and circles, and be especially fitted for subdivision into *scales* adapted to such measurements; and so it is.

By drawing a line GH across the cubit, such as shall separate its right-hand "span" of 5 inches from the remaining 20 inches, a most noteworthy dividing-line is obtained.

The number 5, the central one of the nine digits, was peculiarly hateful to the Egyptians. Sir Gardner Wilkinson says, that, to this day, it is regarded as "the evil number" by the inhabitants of that country. So superstitious are they of this unlucky number, that it is marked by a zero (0) on their watches!

It was, however, a most sacred number to the shepherd kings. These ancient wise men symbolized its importance over and over again, throughout the 5-sided, 5-angled, and wonderfully 5-proportioned, monument they left in Egypt. So, too, the number 5 was of the greatest sacred import to their later kinspeople, the Hebrews.

The whole Jewish polity — religious and civil — was embodied in the five books of Moses. This book of books was kept separate from all other sacred writings, and was even known by the significant name of the "Pentateuch." When Israel left the land of bondage, they marched out five abreast; and, as though to intensify still more their independence of Egyptian superstitions and influences, the Bible tells us they went out "with high hands," themselves symbols of five, raised in the very faces of their former taskmasters.

Now, the cubit number 25 is the perfect square of this sacred number, 5. The dividing line GH we have just drawn through the cubit, cuts off to the right 5 inches, or the *side*, and to the left 20 inches, or the *perimeter*, of this very square; while its *area* is indicated by the number of inches in the whole cubit. These two parts of the cubit, through their equivalents in the pyramid diagram, are respectively equal to $2\pi R$ and to $R'\sqrt{\pi}$.

In these expressions R is the radius of a circle whose *circumference*, and R' the radius of another whose *area*, are respectively those of the special square that we have been constructing. The length of a cubit is thus $2\pi R + R'\sqrt{\pi} = 25''$, in which $R'\sqrt{\pi} = S = 5''$, or the chord of a boss.

Let now the left-hand division of the cubit be subdivided into a scale that shall contain as many units and decimal parts of a unit as there are inches in the radius R, or that of circumferences. It will thus at once become a scale of circumferences.

So, likewise, if the right-hand part be similarly subdivided with respect to R', or the radius of area, a scale of circular areas will result.

Thus new light is shed upon the *nature* of this ancient and honored standard of, not only rectilinear, but of curvilinear, measure. From our present stand-point, the cubit is a rectified pyramid diagram. It is of the primary degree; since its dimensions are those of the boss upon the granite leaf, which we have already seen to be thus primary in all of its relations.

Here, then, we have a standard of linear measure, capable of being subdivided, without wastage, into two such concordant parts that every square and circle in the intimate series of primary lines, unlocked by the pyramid diagram, has thereon its appropriate place and scale.

By means of these scales, having given any other square or circle whatsoever, the workman, by the simple measurement of a side or radius, may instantly obtain all the other square and circular *equivalents* found in the diagram itself, and to that degree of accuracy with which the modern scale-maker shall have graduated its subdivisions.

For instance, let him have a circle of any radius, say $7\frac{1}{2}''$.

To find the side of a square with an equivalent area, he uses the cubit as follows: —

(1) *Rule.* — Go to the scale AB, — that of the areal radius, — and, with a pair of dividers, take off the length of its unit AC. Apply this unit as many ($7\frac{1}{2}$) times to the cubit as there are inches in the radius: the number of inches thus covered will be the length of the *side* of the equal-area square ($13.29''+$).

Again: to find the side ($11.78''$) of a square whose perimeter shall have the same length as the circumference of this given circle.

(2) *Rule.* — Go to the scale DE, — that of the circumference radius, — and, with a pair of dividers, take off the length of its unit DF. Apply this to the cubit as many times as there are

inches in the radius of the given circle: the length thus covered will be the perimeter of the square required, whose side is, of course, one-fourth cf it.

Conversely, having given any *square*, as, for instance, the one whose side is $7\frac{1}{2}$ inches, required its circular equivalents.

First, The circle of equal area.

(3) *Rule.* — Measure the length of its side in terms of the scale *AB*. The result will be the length of the equivalent areal radius in inches ($4.23''+$).

Second, The circle of equal circumference.

(4) *Rule.* — Measure the perimeter of the square in terms of the scale *DE*, or measure one side thereof by this scale, and multiply by four: in either case the result will be the length of the circumferential radius in inches ($4.774''+$).

THE PRIMARY CIRCULAR FUNCTIONS AND THE CUBIT.

One of the most important circles known to practical geometers is the one whose radius is 5, or whose diameter is 10. This circle is so important, that you will find the lengths of its principal parts tabulated in every book of mathematical rules, data, and working formulas. For instance, I have here Haswell's "Pocket-book for Engineers and Mechanics." Upon p. 251 we find tabulated some twenty-five of these principal parts. These values are particularly important because they are the *unit* values, as it were, of the lengths they represent. Knowing them for a circle whose diameter is 10, — the radical of the whole decimal system, — we can, of course, readily determine them for circles whose radii are of any value whatsoever.

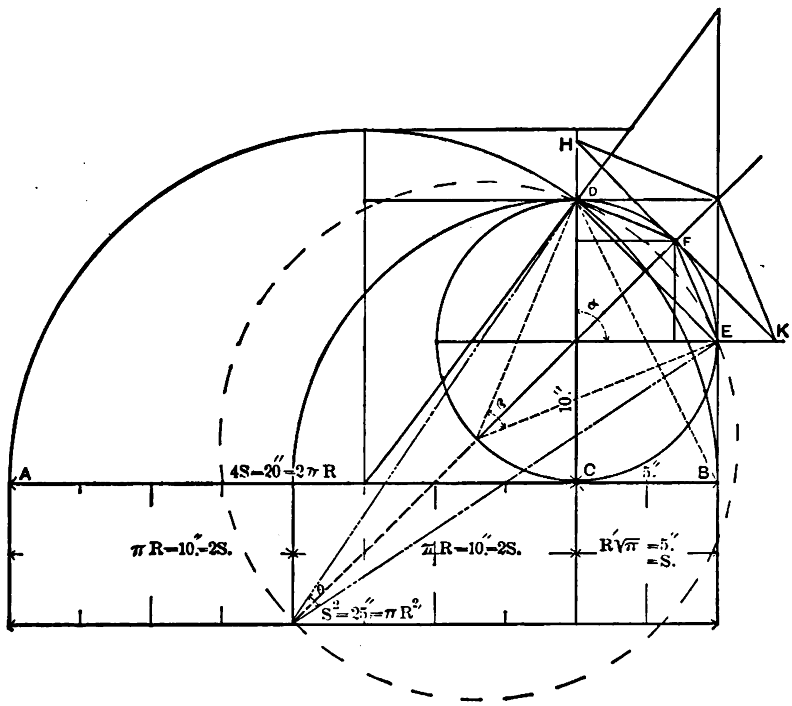
In the course of my pyramid investigations, I have discovered that this circle bears a most intimate and characteristic relation to the sacred cubit, and one which it will be well to note in connection with that branch of our subject which we have just been investigating.

Upon the sacred cubit as a diameter, construct a semicircle.

At the division of the cubit into its 5 and 20 inch parts, — a division that we have already found to be a most notable one for many other reasons, — erect a perpendicular, and produce it until it intersects the semi-circumference just drawn.

This perpendicular is, of course, a mean proportional between $2\pi R$ and $R'\sqrt{\pi}$; that is, between the two main divisions of the cubit. Its length is, therefore, 10 inches, or the diameter of the circle of unit values, to which we have just referred.

Let this circle now be constructed upon its perpendicular



THE CUBIT AND THE PRIMARY CIRCULAR FUNCTIONS.

Fig. 22.

diameter *CD*. Its unique location for the direct comparison of its elements with the other functions of the cubit will be apparent. Each one of the lines represented in the figure is the *absolute* length of a circular function. They are all, in fact, *unit* functions — and *decimal* unit functions, let me reiterate — of the most important character. Their growth from the primary standard of all linear measure — the sacred cubit itself — is thus most beautifully illustrated. Nothing could be more *scientific* than such an arrangement of units as that now before you. They are functions, then, of the sacred cubit as truly as they are those of the radical circle, whose diameter (10) is the base of all modern or decimal arithmetic. Thus, the line DE is the side of an inscribed square; that is, it is the *unit* of such inscribed squares: and the side of a square inscribed in any other circle, as, for instance, the one whose diameter is $47\frac{8}{12}''$, is $4.7\frac{8}{12}$ times the line DE. In a similar way, HK is a unit function of the *cubit*, and the side of a square circumscribed about the primary or 10-inch diameter circle, and DF that of an inscribed octagon, etc. Did the French metrologists, even in their wildest flights of imagination, ever *dream* of such results? What, indeed, can result from the *metre* but confusion? While, on the other hand, what limits dare we place to the benefits that may result to science and mankind, when in the hands of workingmen we place a standard such as this?

THE CUBIT AND SCALES.

By dividing each "unit" inch of the sacred cubit into quarters, the whole cubit becomes scaled off into 100 equal parts: thus, a quarter inch is the one-hundredth of a cubit.

So, likewise, if each quarter inch be redivided into tenths, or, in other words, if the unit inches be divided into fortieths, the cubit becomes a scale of 1,000 equal parts.

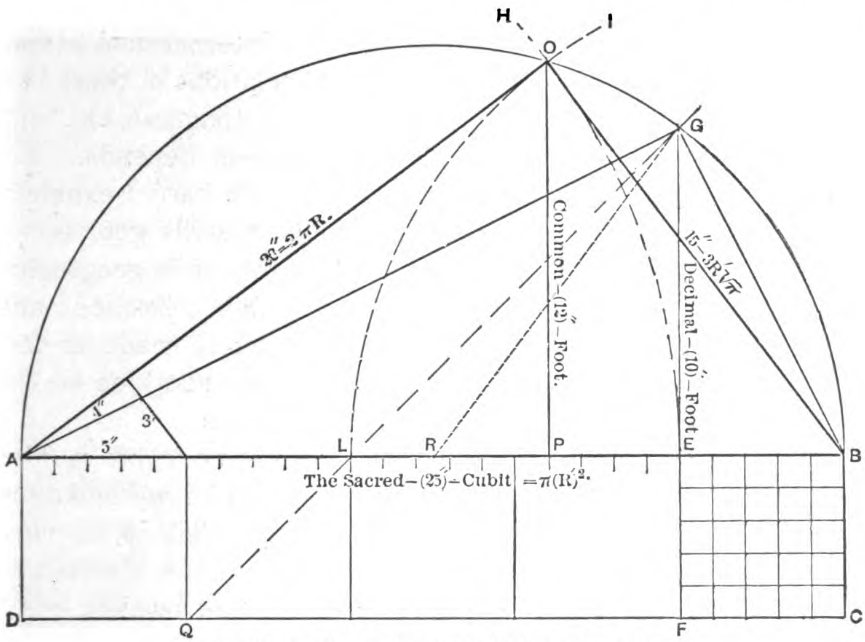
Again, by a division of each "unit" inch into one-hundredths, the cubit becomes a scale of $\frac{1}{2500}$; or, if into one-thousandths, a scale of $\frac{1}{25000}$.

Finally, by the use of micrometers and millimetres (for the deeper purposes of more advanced science), it is manifest that this subdivision may be almost indefinitely continued: thus, by a simple quartering of the last subdivisions, the cubit at once becomes a scale of 100,000.

The fitness of the cubit for decimal scales in general is thus apparent. It is, however, more particularly to be noticed, in this connection, that the various resulting subdivisions of such scales are all well known and ordinary fractions of the unit inch.

THE CUBIT AND THE PYTHAGOREAN TRIANGLE.

Another point of interest connected with this ancient and most sacred metron is its relation to the celebrated "3-4-5 triangle" of Pythagoras, — a triangle whose claims to holding possession of the very foundation of geometry, so far from being shaken, are surer now than ever. My attention was called to this fact by Mr. Jacob M. Clark, C.E., of Elizabeth, N.J., who,



THE CUBIT AND THE PYTHAGOREAN TRIANGLE.

Fig. 23.

upon studying over my diagrams with me, pointed out this and numerous other equally remarkable relations.

Let *ABCD* be a sacred cubit of 25 inches in length, and a span, equal to 5 inches, in width: subdivide it into its square and elementary spans, as in the illustration. Upon *AB* construct a semicircle, and with *A* as a centre, and a radius $AE = 4$ spans, describe the arc *EH:* from *B* as a centre, and a radius $BL = 3$ spans, describe another arc *LI,* intersecting the former upon the semicircle at the point *O.* Join *AO* and *BO:* then will *BOA* form the "3-4-5 triangle," which has the cubit for its hypothenuse. It is through this triangle that the cubit is directly related to the common foot of 12 inches, and also to the Anglo-Saxon yard of three such feet; *for the vertical OP of this triangle is the "common foot."* The decimal foot (10 inches) is, as we have already seen, similarly related to the cubit at its marked division *E*. I shall leave it to Mr. Clark to point out, in due time, the relations of the sacred cubit to the 5-pointed star, or "Pentalpha," — a star which, like the "3-4-5 triangle," comprehends not only the fundamental principles of geometry, but is, *par excellence*, the EMBLEM of the Great Pyramid. In his articles upon "Metric Analogues," in the "International Standard," he has already shown, that, upon the relations of these two figures, the Pentalpha and the triangle of Pythagoras, the only logical subdivision of the circle into degrees depends. He there shows that the proper subdivision into 240° "exhausts the resources, and satisfies all the demands, of strict geometry." The exquisite beauty of such a subdivision for use in geography, astronomy, and chronology cannot be too highly extolled; and the step towards the unification of time lately made in this country will certainly lead in the future to the adoption of the subdivision of the circle proposed by Mr. Clark.[1]

Besides the foregoing connections, the sacred cubit is likewise related to many other important branches of mathematics, such as to the fundamental functions of trigonometry, to many special geometrical principles, and to number in the abstract, to which, for want of space, we can only afford the briefest reference.

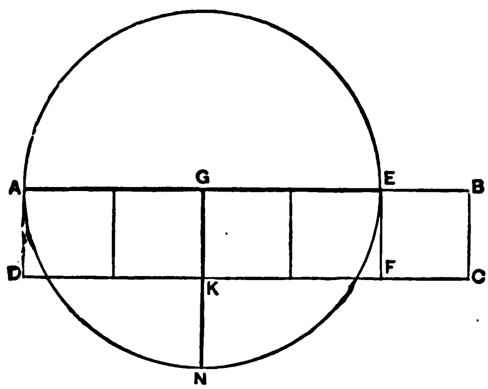
THE CUBIT AND GEOMETRY.

In the practical *geometry* of the arts, the sacred cubit is also of the utmost value to the workman. We can but glance, however, over some of its more salient points; since to exhaust them all, would be to write a treatise upon the Science of Geometry itself. Some of the more noticeable ones are as follows; and, in examining them, it will be observed that they universally depend upon the five-part division, which we have already found to be the basis of so many other important investigations and relations. In the diagrams which we shall now

[1] See Appendix B.

hastily scan, the cubit *ABCD* is divided into only 5 spans: its width is also taken at a span.

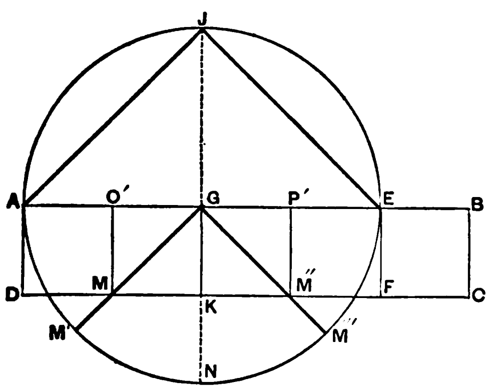
Now, if upon the 20-inch or 4-span segment, as a diameter, we describe a circle, it will be that one whose radius is the decimal foot, or 10 inches. This circle is as important as that one whose diameter itself is the basis (10) of the decimal system of numeration. An examination of this circle in its relations to the cubit reveals the following facts: —



THE POINT, LINE, AND CUBIT.
Fig. 24.

(1) The point *G* falling upon the top of the cubit, and between the second and third spans, gives us our first geometrical idea, — a *point:* from it, within the radius of our conceptions, we are enabled to proceed outwards to the remotest regions of geometry.

(2) Our next geometrical conception is that of the *line:* and, from the diagram, we learn that the shortest line which may be drawn in the circle is that from its point *G* to *N;* i.e., the radius — ten. This line separates the second and third spans without breaking their units, and is also half the longer segment of the cubit. We also perceive that the longest line which may be drawn in the circle, is the one *AE*, likewise passing through the point *G*, and constituting its double radius, diameter, or the full length of the cut-off portion of the cubit.

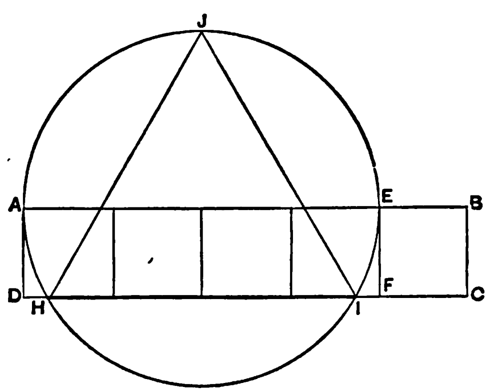


THE CUBIT AND ANGLE.
Fig. 25.

(3) Two lines in their geometrical signification indicate *angle;* and, from the cubit, we learn that $BAD = 90° =$ the

standard angle of geometry, may be constructed by the circle with three important sets of lines. Thus, *AE*, the greatest line in the circle, and the infinitesimal element of the circle at *A*, which indicates the *direction* of its infinite tangent, gives us the radical angle of the cubit at *A*. The lines *AJ* and *EJ*, the two greatest lines which may be inscribed in this circle, also give us this standard angle at the point *J* upon the division line *KG* of the cubit produced to its intersection with the circle. So, too, at *G*, the production of the diagonals *GM* and *GM"*, gives us the radii themselves forming the angle of the perfect square at the centre *G*.

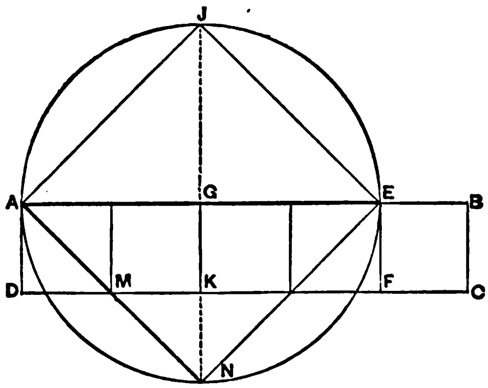
(4) The employment of more than two lines gives us the means of confining space, or of indicating form. In this particular, the cubit is the most wonderful assistance ever discovered. Its ability to comprehend the various regular polygons depends upon its five-part division. This, of course, can be shared by any other scale, since all may be subdivided into fifths; but no other scale than one formed upon twenty-five units, or a due multiple thereof, can be so divided without rupturing the units, nor have as four of its five parts a double radius equal to twenty units. The cubit is, therefore, unique in the *full* realization of the constructions which are to follow.



THE CUBIT AND THE EQUILATERAL.
Fig. 26.

The primary regular form composed of right lines, is the equilateral triangle. The side of this triangle is indicated by that portion *HI* of the base of the cubit which is cut off by the circle on its larger segment.

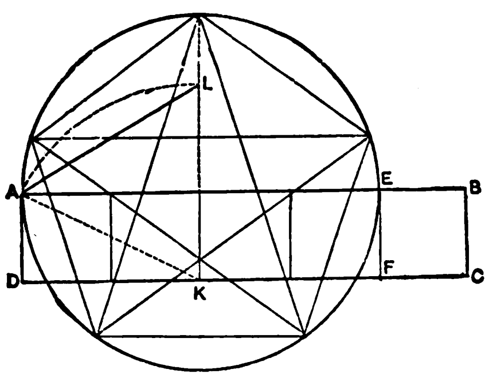
(5) The next of the regular polygons is the square, or tetragon, whose side is the diagonal AM produced to its intersection



THE CUBIT AND THE SQUARE.

Fig. 27.

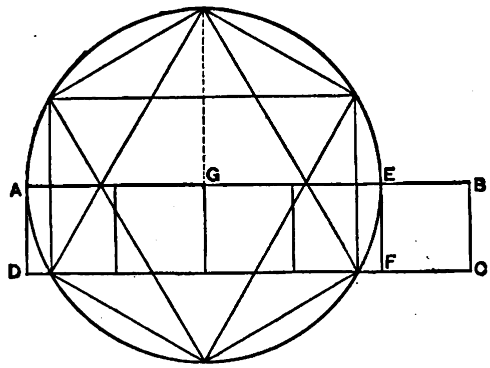
with the circle. It is likewise to be observed, that another important line of the cubit, i.e., AE, is the diagonal of this square.



THE CUBIT AND THE PENTALPHA.

Fig. 28.

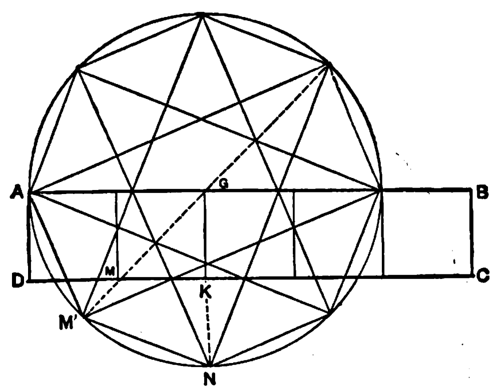
(6) To form the pentagon with the cubit, take K as a centre, and the diagonal KA as a radius; describe the arc AL; then will the chord AL be the side of a pentagon.



THE CUBIT AND THE HEXAGON.

Fig. 29.

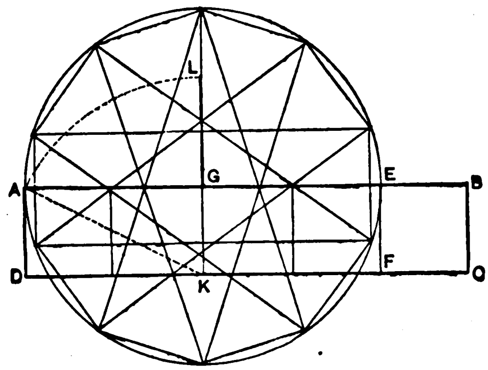
(7) The side of a hexagon is, of course, the radius or distance $AG = 10''$, or to half the cut-off portion, AE, of the cubit.



THE CUBIT AND THE OCTAGON.

Fig. 30.

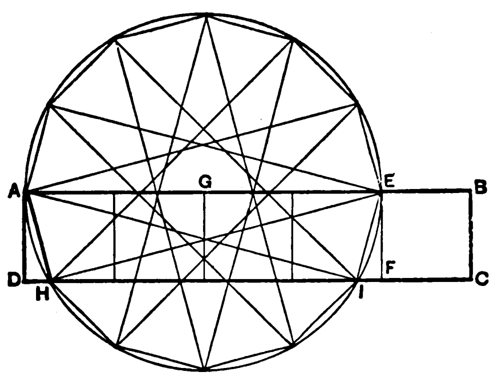
(8) To obtain the octagon, produce the diagonal GM to M'; join this with the point A, or produce the cubit division GK, till it also intersects the circle; then will NM also be the side of the required polygon.



THE CUBIT AND THE DECAGON.

Fig. 31.

(9) The side of the decagon is the distance from *L* (already determined in section 6) back to the cubit, or *LG*.

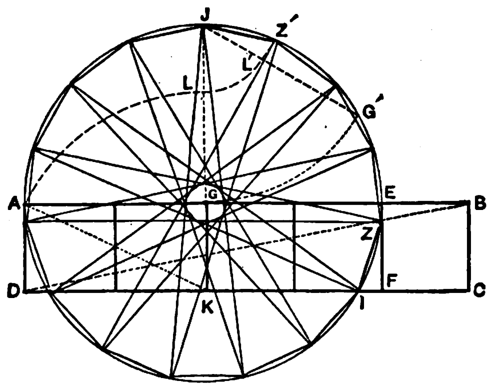


THE CUBIT AND THE DUODECAGON.

Fig. 32.

(10) The side of the duodecagon is the chord width *AH* of the cubit, determined by joining the points *A* and *H*, in which the circle intersects it.

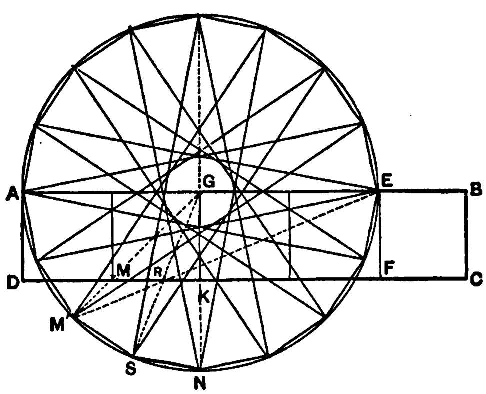
(11) To obtain the 15-sided polygon, join the points D and B, i.e., draw the full diagonal of the cubit; then connect the point Z, in which this line intersects the circle, with the point



THE CUBIT AND THE 15-SIDED POLYGON.

Fig 33.

I, in which the latter intersects the base DC of the cubit: it is the side of the polygon required. Or, from the point J lay off the chord JG' equal to the radius AG, and from G' lay back the chord $G'Z'$ equal to GL the side of the decagon: then will JZ' be the chord of one-fifteenth of the circumference ($\frac{1}{6} - \frac{1}{10} = \frac{1}{15}$), or the side of the required polygon.



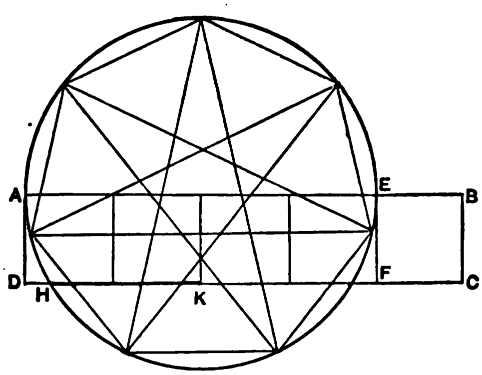
THE CUBIT AND THE 16-SIDED POLYGON.

Fig. 34.

(12) To determine the 16-sided polygon, produce the diagonal GM of the cubit to M', where it intersects the circle; draw $M'E$

intersecting the base of the cubit at R; through G draw GR, and produce it to S; likewise produce GK to N. Join NS, and it is the side of the required polygon.

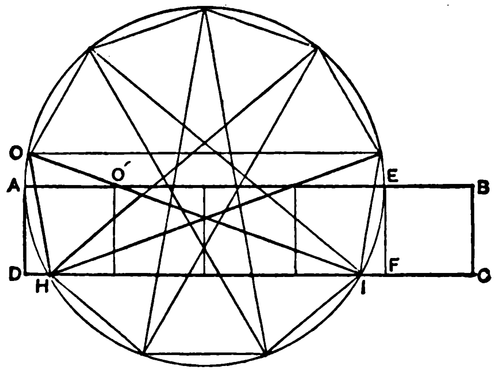
The foregoing comprise all those elementary polygons that have hitherto been classified as *rational*, since they may be obtained by a direct geometrical process. Those left out of the series, the heptagon or 7-sided, the nonagon or 9-sided, the hendecagon or 11-sided, the 13-sided and the 14-sided polygons, are termed *irrational*, since they have not hitherto been so obtained. An examination of the peculiar properties and power of the sacred cubit reveals the fact, however, that not only may the rational polygons and their corresponding stars be determined directly from it, as we have shown above, but also, that, by its all-potent means, a series of most remarkable working-approximations to the irrational ones may be arrived at. Thus,—



THE CUBIT AND THE HEPTAGON.

Fig. 35.

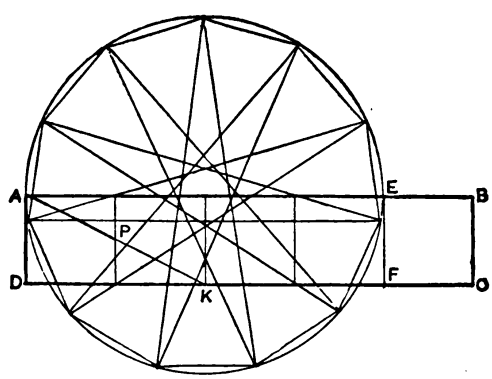
(13) To obtain the heptagon, employ HK as a chord (i.e., half of that portion of the base of the cubit cut off by the circle), and it will be the side of the required polygon true to thousandths, or to the third place of decimals.



THE CUBIT AND THE NONAGON.

Fig. 36.

(14) To obtain the nonagon, join *I* and *O′*, and produce the oblique line, so obtained, until it intersects the circle at *O*. Join *O* and *H*, and it is the chord required, likewise true to the decimal .00205.

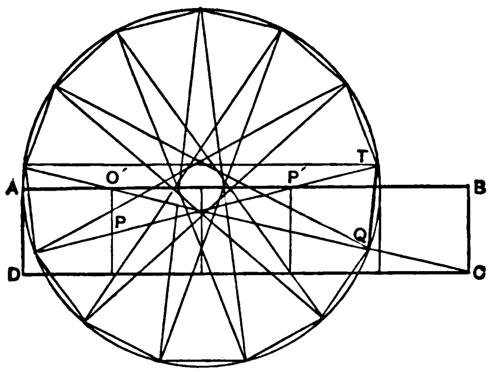


THE CUBIT AND THE HENDECAGON.

Fig. 37.

(15) To obtain the hendecagon, draw the diagonal from *K* to *A*; and one-half of it, *PA*, is the closest approximation known to the chord required.

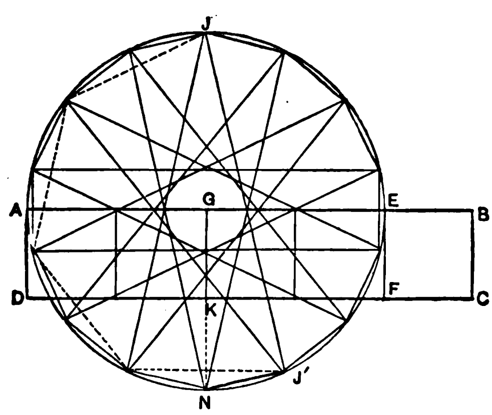
(16) For the 13-sided polygon, draw the diagonal from *C* to *O′*, and through *P* and *P′* produce the diagonal *PP′* to *T*, its



THE CUBIT AND THE 13-SIDED POLYGON.

Fig. 38.

intersection with the circle: then will *QT* be the side required to its nearest geometrical construction.



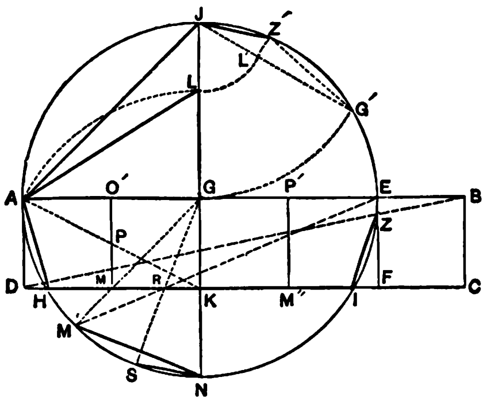
THE CUBIT AND THE 14-SIDED POLYGON.

Fig. 39.

(17) For the 14-sided polygon, lay off the distance *HK* four times as a chord, commencing at *J*, and terminating at *J′*. Produce *GK* to *N*. Join *NJ′*: it is the chord required. Or (see Fig. 41), from *H* as a centre, and with *HK* as a radius, describe the arc *KY*; produce *FE* to *U*, making *EU* equal to *EB*; through the point *U* thus determined, and *V*, the middle point of *GK*, draw the line *UV*, which intersects the base of the

cubit at *W.* Draw *GW*, and produce it till it intersects the circle at *X.* Join *XY:* it is the chord required.

The foregoing constructions, dependent upon the five-part division of the cubit, as related to the circle whose radius is 10, struck upon its longer segment *AE* as a diameter, are represented collectively in the two following diagrams.

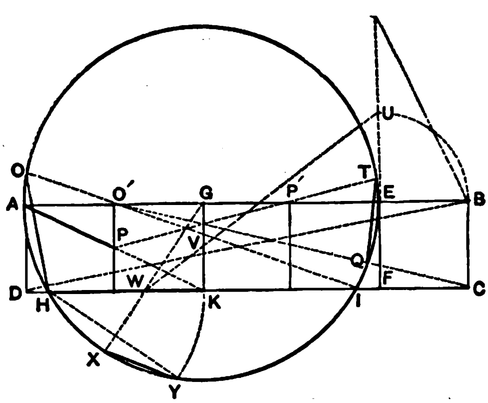


THE CUBIT AND RATIONAL POLYGONS.

Fig. 40.

Fig. 40 gives us the construction of the several rational polygons as follows: *ABCD* is the cubit; *E*, *P′*, *G*, and *O′* its points of division into spans or fifths; *JINH* the circle upon its larger segment *A E* as a diameter. To recapitulate, therefore, *HI* is the side of the equilateral triangle, *AJ* or *AN* that of the square or tetragon, *AL* that of the pentagon, *AG* of the hexagon, *NM′* of the octagon, *GL* that of the decagon, *AH* that of the duodecagon, *JZ′* or *IZ* that of the 15-sided, and *NS* that of the 16-sided, polygon.

In Fig. 41, constructed and lettered to correspond to the foregoing, and to all the preceding several diagrams under this heading, *HK* is the side of the heptagon, *HO* that of the nonagon, *AP* that of the hendecagon, *QT* that of the 13-sided, and *XY* that of the 14-sided, polygon.



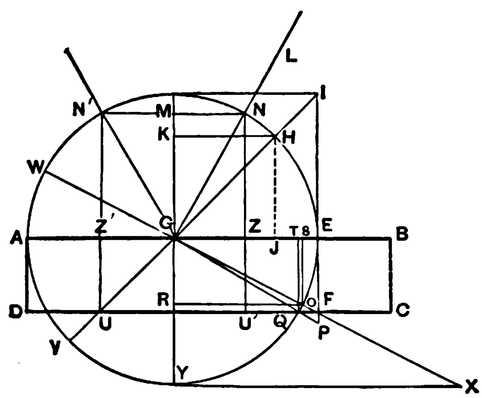
THE CUBIT AND IRRATIONAL POLYGONS.

Fig. 41.

From these primary polygons, by proper bisections, the whole series of regular geometric forms which result from them may be duly constructed, and throughout the process the *power* of the cubit itself, as a means towards the end, be constantly felt.

THE CUBIT AND TRIGONOMETRY.

The cubit is likewise directly connected with trigonometry through the circle described upon its longer segment (20″) as a diameter. This, as above, is the circle whose radius is numerically the decimal root 10. To illustrate this, let, as before, *ABCD* be a cubit, and *EF* its extreme five-part division; upon the longer segment *AE* as a diameter construct a circle, and let the radical functions of trigonometry be drawn therein; then will their individual relations to the cubit be severally apparent, as also the convenience of the position of the cubit itself for measuring them thereon, either in radius or in inches,—the unit of the cubit itself. For instance, *EI* being equal to radius, or to 10, the angle *HGE* is an octant, or is 30° on a scale of 240° to the whole circle; i.e., 45° to the circle of 360°, which is its present subdivision as derived from Babylon. Its sine is *HJ* equal to its cosine *HK*, the measure of which latter is *GJ* upon the cubit. Now, it is manifest, that to whatever degree of minuteness and accuracy our cubit itself is subdivided, be it to twenty-fifths, or to millionths, to that same degree of accuracy, without tables or formulas, may the practical workman obtain the numerical values of the circular functions he may need by a direct appeal to the cubit itself. Thus, let it be supposed that a workman desires to obtain the angle whose cosine is $\frac{1}{2}R$, or 5″: he will produce the cubit division $U'Z$ to N; then will $NM = 5''$, or $\frac{1}{2}R$, and be the cosine of the angle NGB required. In a similar way, the cubit subdivision UZ' produced to its intersection N' with the circle, gives us the angle $N'GB$, whose cosine is $-\frac{1}{2}R$ or $-5''$, etc., ranging throughout the



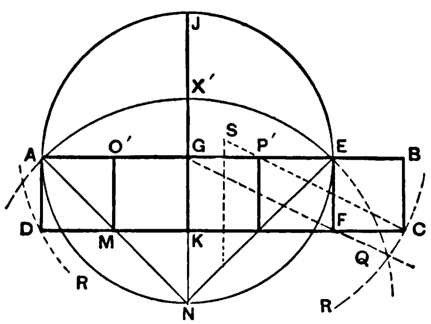
THE CUBIT AND TRIGONOMETRY.

Fig. 42.

whole scope of this branch of mathematics. Should the radius of some circle be any thing else than 10, the decimal root, the cubit still affords us the direct means of obtaining its trigonometrical angles and functions; since, for instance, one whose radius is $7\frac{1}{4}$ inches has for its cosine — $\frac{1}{2}R$, a length equal to .725 that of the 10″ radius: in other words, a new circle centred at G, and with a radius 7.25″, will limit all the lines and functions at their new points, and determine the same angles.

THE CUBIT AND NUMBER IN THE ABSTRACT.

There is another illustration, which shows still further the connection of the cubit, as a *radical metron*, with all mathematics. In the whole range of number in the abstract, as applied to the expression of the areas and circumferences of circles, there is but one circle whose numerical expressions for area and circumference are identical: this is the circle whose radius is 2. The area πR^2 of such a circle is $\pi(2)^2 = 4\pi$. Its circumference, $2\pi R = 2\pi(2)$, is also 4π.



THE CUBIT AND ABSTRACT NUMBER.

Fig. 43.

Thus the same sequence of figures, to infinity, expresses a number of square units in the one, and of linear units in the other; to wit, $4\pi = 12.56636 +$. Now, an examination of the larger segment of the cubit, after making the important or five-part division, reveals the fact, that this segment is 4 in terms of the span, and hence that the circle upon it as a diameter, and whose utility for polygonal constructions we have already discussed, is, in effect, that one whose area is equal to its circumference in terms of the cubit fifths or spans. Fig. 43 illustrates the properties of this circle. Its area = its circumference = respectively 12.56636 + square and linear spans. Hence the arc $AJ = 3.141592 +$ spans; the semi-circumference $AJE =$ 3.141592 +, etc., decimal feet; and the full circumference = 3.141592 +, etc., times the larger segment of the cubit, or is equal to 15.707960 +, etc., palms of 4″ each, etc. So, too, the sector $AGJA = 3.14159 +$ square spans; the semicircle $=$ 3.141592 + decimal square feet, etc.

Another remarkable numerical property of this circle is the fact that the area of the lune $AX'EJA$, being equal to that of the triangle AEN, is exactly 100″, or 4 square spans ($= (10'')^2$); hence the semilune $EX'JE = 2$ square spans, or 50″; or, since the quadrant EGJ, as already shown, is equal to 3.141592 +, etc., square spans, the semicircular segment EGX' is equal in area to 1.3141592 +, etc., such units; and the circular sector $NX'EN = 3.141592$ square spans; that is, the octant of the circle whose centre is N, and whose radius is NX', is equal to the quadrant of the one whose centre is G, and whose radius is 2 spans. Now, the radius $NE = 2\sqrt{2}$ spans is a remarkably close working-approximation to that of the circle whose area is a square cubit; it being 2.8284 +, etc. (say, 2.83), spans, against the required π value, 2.8209 (say, 2.821) spans, — values which differ by only .009 of a span. This construction thus affords the practical workman another wonderfully close means of approximating to the square of a circle, and one that is intimately related to that depending upon the five-part division which we have already studied.

It is worthy of note, too, in this connection, to observe that these two working-approximations to the square of the circle — the one from S with a radius SC, and the other from N with a radius NE — intersect each other at Q, upon the cubit diagonal GF produced.

PERSONAL REFERENCES OF THE CUBIT.

As it is *earth*-commensuric, so, too, the cubit is also peculiarly commensuric with the *human frame*. This is most fortunate for the poor man, and for emergencies.

Its "unit," — the "pouce," or "polgida" as it is sometimes called in other countries, — that is, the "thumb," or *inch*, is the thumb-breadth of an able-bodied man; while the entire standard, as its name (cubit) implies, is such a man's arm-length.

Surely no one is so poor as not to have a fair average of these measures always at his hand and side, wherewith to govern both his life and work. In the true canon of human proportions founded upon the π relations of the pyramid diagram, and drawn at its actual scale, we doubt not that the inch and cubit will be found to be the unit and standard in terms of which all of the principal parts will be commensurable.

THE TRUE CANON OF HUMAN PROPORTIONS.

We have already had occasion to coin the word *mesocosm*, as one specially applicable to the Great Pyramid in its intermediate position to man and to the universe: it is at once an image of the microcosm and the macrocosm.

We need not here review the discoveries of others which go to show the intimate cosmic references of the pyramid to the earth itself, and to the universe, but, taking it for granted that all are more or less familiar with these references, shall confine ourselves to an examination of the proportions of the human figure as viewed in the light of the pyramid diagram.

In his elaborate work upon the ancient and modern systems of proportion, the sculptor, W. W. Story, is led to propose, as the basis of a new canon, a circle circumscribing an equilateral triangle and a square. From the relations between the right lines in these several figures, he determines those between the various parts of the human frame. The primary rule in this canon is thus enunciated by Mr. Story: "The entire height of the figure is 3½ diameters, or 7 radii; 4 times the base of the triangle, or 5 times the side of the square," and he finds that this diagram "will satisfy all the scientific and practical conditions demanded of such a canon." "It gives not only the lengths of the body, but also the breadths of the body viewed in front, and the depths as viewed in profile." Mr. Story then enumerates and explains the various measures of the body founded upon this canon.

At the close of his work, however, we notice the following very important admission, and one which cannot but be fatal to the strict accuracy of his rule of proportion. "In the foregoing pages, 4 times the side of the triangle, 7 times the radius, and 5 times the side of the square, are sometimes treated as equal. This, however, in a rigid geometric sense, is not true.

Four times the side of the triangle is a fraction less than 7 radii, and 5 times the side of the square is a fraction greater than 7 radii. The equations —

$$T = \tfrac{7}{4}R$$
$$S = \tfrac{7}{5}R,$$

however, though only approximative, are not only sufficiently exact for all practical purposes (the error in each case being greater than $\frac{1}{100}$, but less than $\frac{2}{100}$ of the radius), but also are approximations of the most scientific kind; the true equations being —

$$T = \sqrt{3}R$$
$$S = \sqrt{2}R,$$

and $\frac{7}{4}$ and $\frac{7}{5}$ being convergents to $\sqrt{3}$ and $\sqrt{2}$ respectively."

Now, as a necessary matter of fact, the human form is not made and proportioned upon any mere system of approximations, but upon some diagram of the most rigidly correct order. For all the practical purposes of the ordinary workingman, Mr. Story's canon will undoubtedly answer; but his own admissions show us, that, for the *ideal*, we must look higher, and actually include these very π proportions, if we hope to attain the goal. But to introduce these is to admit the whole pyramid system through the same gate; for Mr. Story's canon would then become but a different form of the pyramid diagram, — a special case.

Now, the celebrated canon of Polycletus was the recognized standard of the ancients; and it is universally admitted that it came, at least indirectly, from Egypt. "It is to Egypt," says Story, "that the formal and rigid divisions of all the early Greek and Etruscan statues plainly point;" for "the Egyptians possessed a canon of exact and geometric character."

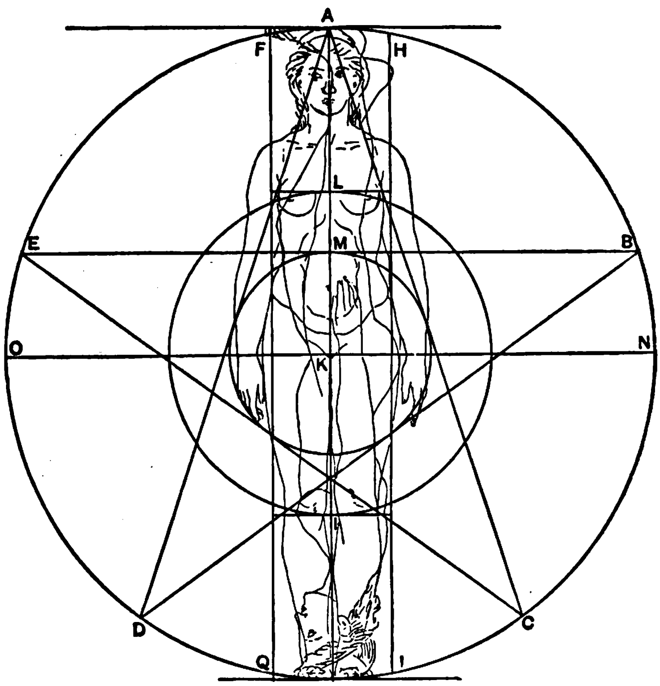
"The Greeks," says Diodorus, "had no knowledge of this kind of art; though the use of it was perfectly known to the Egyptians. For the Egyptians did not measure with the eye the composition of their statues, but with measures; so that out of many blocks, worked according to an exact and fixed measure, they brought the statue to its perfection: and what may be called truly wonderful is, that various artists in different places should, upon a certain and established measure, compose a single statue in twenty, and sometimes forty, pieces, which would all exactly agree." Every thing in this statement of Diodorus points to a Promethean system, and eliminates from it all that is Epimethean and haphazard. It savors of the true masonic method, so intensified at Gizeh and Jerusalem, where the work grew towards its completion without the sound of hammer or of iron tool.

Such a method must have been founded upon the strictest principles of geometry; and, if the canon of Polycletus was a perfect one, it, too, must have come up out of Egypt with circummetric proportions. "The canon of Polycletus has, however, been lost, and with it the *Doryphoros*, which was its practical embodiment. The only vestige of it that we possess, is to be found in the pages of Vitruvius, who, writing upon architecture, gives us, incidentally, a confused account of some of its principal features."

One of the rules preserved by Vitruvius is as follows: "So also the navel is the natural centre of the body; for if a man be placed supine, with his hands and feet spread, and the navel be taken as the centre of a circle, the circumference would touch the extremities of the fingers and feet." But this statement of Vitruvius is evidently in error; for the pubis, and not the navel, is the natural and actual centre of the human figure. Mr. Story himself doubts the accuracy with which Vitruvius enunciates the principles of Polycletus; for he says, "It is more than doubtful whether he really understood the canon; as he was not a sculptor, and would have had little practical need to study it."

Now, as a matter of fact, let us examine the human figure under true pyramidal proportions, and see what this principle of Polycletus actually was. Ballard has shown that the Pentalpha is the emblem of the Great Pyramid. Inscribe it, therefore, together with an erect human figure, in a circle. The centre of the star and circle will be found at the pubis, and not at the navel; and, if the arms and legs be spread out easily, the whole figure will have its five extremities found to coincide with the several corresponding points of the star. The latter is thus not only discovered to be an accurate and mathematical emblem of the five-sided and five-pointed π pyramid, but a reason is seen for the tradition, that, from time immemorial, has made it the mystic symbol of the human form divine.

In the figure, it will also be noticed that the natural division into quarters is borne out; the centre of the breast being midway between the pubis and the crown of the head, and the base of the patella midway from the pubis to the sole of the foot.



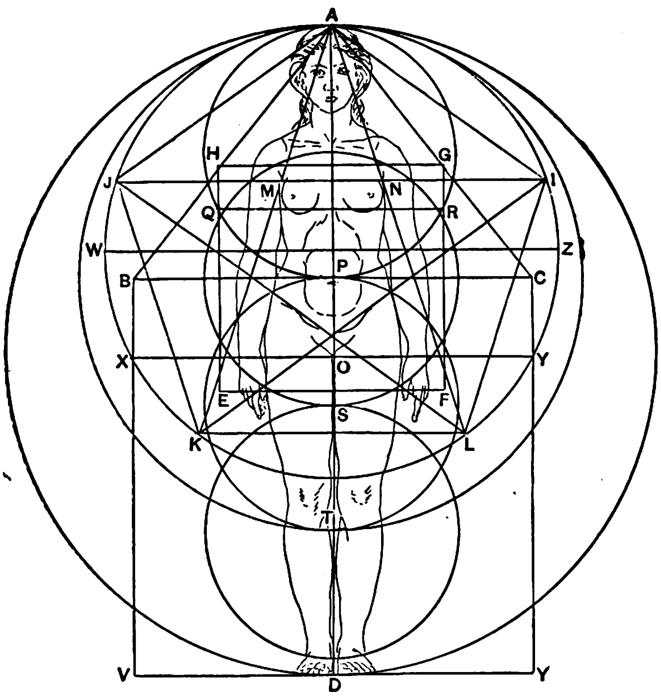
THE PENTALPHA, THE PYRAMID, AND THE CANON OF PROPORTION.

Fig. 44.

In order to develop these facts more clearly, it will be noticed that an inverted profile of the human form is put upon the same diagram.

Vitruvius also gives another rule of the Polycletian canon, and in rather more accurate language. He says, "As the scheme of the circle is found in the body, so also is the scheme of the square; for if the measure be taken from the soles of the feet to the top of the skull, and applied to the hands outstretched on either side, it will be found that the width and height are equal, so as to form a perfect square."

Now, this statement verifies upon the figure which we have given, but not upon the one described from the navel as a centre. For if the circle of our own diagram be enclosed in a circumscribing square, and the arms be outstretched horizontally, the tips of the fingers will just come to the vertical sides of the square. Thus the two, and only, rules of the celebrated canon of Polycletus which have been preserved to us are actually verified



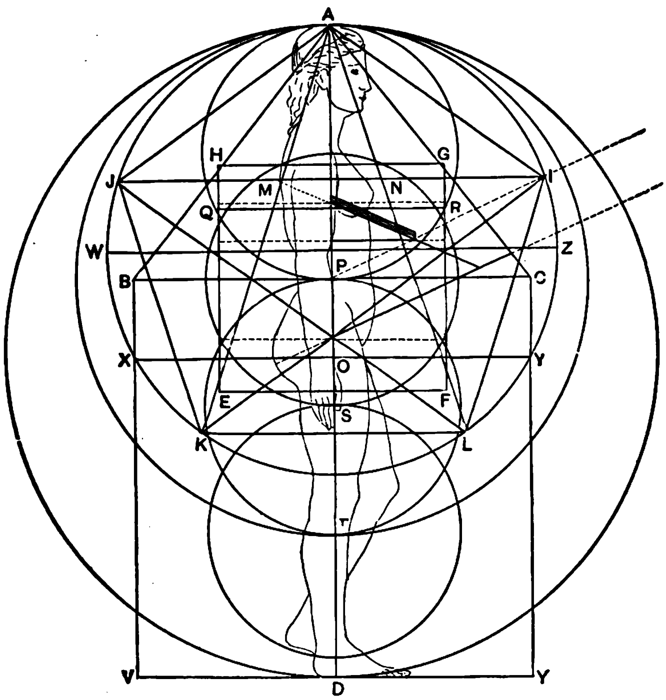
THE HUMAN FIGURE AND THE PYRAMID DIAGRAM.

Fig. 45.

by the most primary application of the true pyramid proportions.

Let us now apply the full pyramid diagram to the human form. Taking the centre, as before, at the pubis, and standing the figure upon the lower side of the pyramid's square base, let the crown of its head come to the vertex of the capstone. It will be at once noticed that a host of important and mathematical coincidences result. Thus, to mention but a few, the line *XY*—the intersection of the circle of equal area with the base of the pyramid—is that which divides the figure in twain; the small of the body, or waist, is on the line *BC*, which is the upper side of the square base; and all the intersections of the various lines of the diagram mark important points, from which the measures of a true canon may be fully realized.

In Fig. 46 we give a side view of the human form against the same diagram, and, in addition, have introduced the general directions of the several known passage-ways of the pyramid, to show that they locate respectively the womb, the heart, and the lungs. We cannot here examine these diagrams at greater



THE PYRAMID DIAGRAM AND THE HUMAN FORM.

Fig. 46.

length. That they comprehend the true principles of the lost canon, however, we are fully persuaded; and we shall leave it for others to develop its elementary proportions.

It is manifest, therefore, that if a perfectly proportioned human figure be drawn at a height of 10″, and upon its front and profile views the pyramid diagram be laid, it will enable us to reproduce, at any scale, the true proportions of the God-like image; and there can be little doubt but that the same wisdom which comprehended the grandeur of the macrocosm in the wonderful proportions of the Great Pyramid, failed not to grasp the lines of beauty of the microcosm.

The cubit is but another form of the mathematical relations of the boss upon the granite leaf, as exemplified by the pyramid diagram; and thus the cubit itself comprehends all of the relations, so intimate and far-reaching, which we have hastily reviewed.

Possessed of such a standard, then, and initiated into its higher uses, the ordinary "workman" is rendered practically independent of π values and mathematical tables. With it, he can readily construct at their true dimensions any of the simpler "units" that he is in the habit of employing; while in it alone the advanced student, the architect, the sculptor, the astronomer, and the metrologist, can find the very ideal of linear measures.

Such relations as we have noted will be sought for in vain at the bar of every other standard of linear measure. We defy the advocates of the "metric system" to bring forth such facts as these with the wand by which they wish to make us "change our times and seasons." The "sacred cubit," like the "rod which Aaron" flung upon the sands of Egypt centuries ago, shall literally swallow up the serpent of "science falsely so called." In the bright future of universal metrology, the false, unscientific standard of an age which was the very nightmare of injustice and infidelity, cannot exist. The lying magician may indeed perform a wonder, but his *wand* shall be devoured.

The cubit is *unique* among measures. It cannot be improved upon. It is the least common multiple of all that we can desire in a linear standard, while its unit inch is the greatest common divisor of things man has to measure.

THE UNIVERSAL PROBLEM.

In the winter of 1881, while generally investigating the mathematics of our subject, I had occasion to propose quite an interesting problem in geometry to several of the army-officers serving at the Presidio. This problem was, to construct an equilateral triangle whose area should be equal to that of a given square.

It was necessary to obtain a simple solution of this question as a step in certain pyramid investigations which I then had on hand, and several very ingenious ones were soon proposed.

Of course, we shall not have time to examine these several solutions: they are foreign to our immediate subject. I shall ask your attention, however, to one of them, and to this one, not only because it touches so closely upon that subject, but more particularly because I discovered it with the assistance of the already wonderful "pyramid diagram."

The principles involved in this solution are so general, that their application gives rise to the most universal geometric method you will probably ever yet have heard enunciated. On this account, I shall call it "The Universal Problem."

We shall probably arrive at a better understanding of this problem, and of its special application to the case in point, by a short preliminary course of reasoning.

A thorough comprehension of the *principles* involved in the now familiar pyramid diagram, has already shown us that such a direct relation certainly exists between circles and squares as to prove that their mutual quadrature and rectification is an undoubted possibility.

These principles may be translated into the following geometric enunciation or theorem: If the perimeter of any square be put into the form of a circle, and another square be constructed whose area shall be equal to that of this circle, then will the area of a second circle, whose diameter is equal to the side of this second square, be equal to the area of the original square.

But if these relations which we have discovered be true of circles and *squares*, they must, of course, be true of circles, and all other regular rectilinear figures whatsoever; since all such figures can be recast into equal squares, and *vice versa.*

Investigation not only fully established this proposition, but afforded an analytical demonstration of its accuracy.

Reduction of Circular Areas to any other "Regular" Form.

General Demonstration.

Let $\pi R^2 =$ the area, and $2\pi R$ the circumference, of any circle. Denote by P any regular polygon of n sides, the total perimeter of which shall equal $2\pi R$.

Let S denote one of these sides, and let $b =$ its apothegm; then area of $P = 2\pi R \frac{b}{2} = \pi Rb$.

Let there now be *another* circle, whose area $\pi(R')^2 =$ the area of $P = \pi Rb$; then $R' = \sqrt{Rb}$.

Circumscribe about this latter circle a new polygon P' of n sides; denote its area by A'. Then, since the polygons P and P' are similar, we shall have $\pi Rb : A' :: b^2 : (\sqrt{Rb})^2$; i.e., (the apothegm of $P')^2$.

Therefore, $A'b^2 = \pi R^2 b^2$, or $A' = \pi R^2$; i.e., the last polygon P' of n sides will have the same area as the original circle.

Q.E.D.

Example. — Special application of the above, in the case of a circular area put into the equilateral triangular form: —

$\pi R^2 =$ area of given circle, and $2\pi R$ its circumference.

$P =$ an equilateral triangle whose perimeter $= 2\pi R$.

S, then, $= \frac{2}{3}\pi R$; altitude $= \frac{1}{3}\pi R\sqrt{3}$; apothegm $= \frac{1}{3}$ altitude $= \frac{1}{9}\pi R\sqrt{3}$.

Area $P = \frac{1}{2}(\text{base} \times \text{alt.}) = (\frac{1}{3}\pi R)(\frac{1}{3}\pi R\sqrt{3}) = \frac{1}{9}\pi^2 R^2\sqrt{3}$.

Let, now, second circle be discussed: $\pi(R')^2 = \frac{1}{9}\pi^2 R^2\sqrt{3}$.

$R' \therefore = \sqrt{\frac{1}{9}\pi R^2\sqrt{3}} = \frac{1}{3}\sqrt{\pi}R\sqrt{\sqrt{3}}$. But, since P' is to be *circumscribed* about this circle, the altitude of P' will equal 3 times R', or $= \sqrt{\pi}R\sqrt{\sqrt{3}}$, and its apothegm equals R' itself; hence

$$\tfrac{1}{9}\pi^2 R^2\sqrt{3} : A' :: (\tfrac{1}{9}\pi R\sqrt{3})^2 : (R')^2,$$

or

$$(\sqrt{\pi}R\sqrt{\sqrt{3}})^2;$$

hence

$$A'\tfrac{3}{81}\pi^2 R^2 = (\tfrac{1}{9}\pi^2 R^2\sqrt{3})\,(\pi R^2\sqrt{3}),$$

and

$$A' = \frac{\frac{(\pi^2 R^2\sqrt{3})(\pi R^2\sqrt{3})}{9}}{\frac{\pi^2 R^2}{27}};$$

or, reducing,

$$A' = \pi R^2. \qquad \text{Q.E.D.}$$

Further investigation showed that the principle applied to all regular polygons whatsoever, and also afforded an algebraic demonstration of this fact.

I shall now try and enunciate the more general proposition, and show its application, by solving thereby the problem proposed last winter.

The general theorem is as follows: If a regular polygon (P') of any number (n') of sides be so constructed that its perimeter shall be equal to that of a given regular polygon (P) of n sides, and a third polygon (P'') be constructed similar to P, but of the same area as P', *then, the area* of a fourth polygon (P'''), similar to P', and circumscribed with P'' about the same circle (i.e., of equal apothegm), will be equal to that of the original polygon P.

Let P be a given regular polygon of n sides. Let a equal the length of one of its sides, and b its apothegm. Then will its area be equal to $\frac{anb}{2}$, and its perimeter equal to an.

Let P' of n' sides represent another regular polygon of equal perimeter (an) to P, and let its apothegm be denoted by b'. Then

$$S' = \text{a side of } P' = \frac{an}{n'},$$

and

$$A' = \text{area of } P' = an\frac{b'}{2}.$$

Construct now a third regular polygon P'' of n sides (i.e., similar to P), but whose area $= A'' = \frac{anb'}{2}$ is the same as that of P'. Then, since P and P'' are similar, we have,

$$an\frac{b}{2} : an\frac{b'}{2} :: (b)^2 : (b'')^2,$$

or

$$\frac{anb(b'')^2}{2} = \frac{anb'(b)^2}{2};$$

therefore

$$b'' = \sqrt{bb'}.$$

And since $\frac{1}{2}$ (perimeter of P'')$\sqrt{bb'}$ = area $A'' = \frac{anb'}{2}$, we have

$$\text{Perimeter of } P'' = \frac{anb'}{\sqrt{bb'}} = \frac{an\sqrt{b'}}{\sqrt{b}},$$

and

$$S'' = \text{a side of } P'' = \frac{a\sqrt{b'}}{\sqrt{b}} = a\sqrt{\frac{b'}{b}}.$$

Let now P''', a polygon of n' sides, and similar to P', be constructed, but with an apothegm equal to that of P''; i.e., equal to $b'' = \sqrt{bb'}$. (P'' and P''' can thus be circumscribed about the same circle.) Now, since P' and P''' are *similar*, we have,

$$\text{Area } P' : \text{area } P''' :: (\text{apothegm } P')^2 : (\text{apothegm } P''')^2,$$

or

$$\frac{anb'}{2} : A''' :: (b')^2 : (\sqrt{bb'})^2;$$

that is,

$$A'''(b')^2 = \frac{anb'}{2}(\sqrt{bb'})^2;$$

therefore

$$A''' = \text{area of } P''' = \frac{\frac{anb'}{2}(\sqrt{bb'})^2}{(b')^2} = \frac{anb}{2}. \qquad \text{Q.E.D.}$$

Example. — Take a square area equal to a^2, and let it be required to pass it, by the above method, into the form of an equilateral triangle.

Given square. — $S = a$, $A = a^2$, perimeter $= 4a$.

First triangle. — Perimeter $= 4a$, $\therefore S' = \frac{4}{3}a$, altitude $= \frac{2}{3}a\sqrt{3}$, area $= \frac{4}{9}a^2\sqrt{3}$.

Second square. — Area $= \frac{4}{9}a^2\sqrt{3}$, $S' = \sqrt{\text{area}} = \frac{2}{3}a\sqrt{\sqrt{3}}$; hence the radius of its inscribed circle $= \frac{1}{2}S = \frac{1}{3}a\sqrt{\sqrt{3}}$.

Second and final triangle. — Since this triangle is to be circumscribed about the same circle as the second square, its apothegm equals the radius of the latter; i.e., $= \frac{1}{3}a\sqrt{\sqrt{3}}$. The altitude of a regular triangle equals 3 times its apothegm, equals $a\sqrt{\sqrt{3}}$; and since the altitude of an equilateral triangle $\times \frac{2}{\sqrt{3}} =$ a side thereof, we have

$$S'' = \frac{2a\sqrt{\sqrt{3}}}{\sqrt{3}} = \frac{2a\sqrt{\sqrt{3}}}{\sqrt{\sqrt{3}}\sqrt{\sqrt{3}}} = \frac{2a}{\sqrt{\sqrt{3}}}.$$

$\therefore$ the area of this triangle $= \frac{1}{2}(\text{alt.} \times \text{base}) = \frac{1}{2}\left(a\sqrt{\sqrt{3}} \times \frac{2a}{\sqrt{\sqrt{3}}}\right) = a^2$.

Q.E.D.

Application of the above to the square whose side is $10 = a$. The area of the required triangle is thus $100 = a^2$. Applying the formulas of the final triangle above, we have

$$S'' = \frac{2a}{\sqrt{\sqrt{3}}} = \frac{20}{1.316074+} = 20 \times (.7598+) = 15.1960+, \text{ etc.}$$

$$\text{Alt.} = a\sqrt{\sqrt{3}} = 10(1.3160740+, \text{ etc.}) = 13.160740+, \text{ etc.}$$

$$\text{Area} \therefore = \tfrac{1}{2}(S'' \times \text{alt.}) = \tfrac{1}{2}(15.1960 \times 13.160740+, \text{ etc.})$$
$$= 99.99+, \text{ etc.}, = 100.$$

Q.E.D.

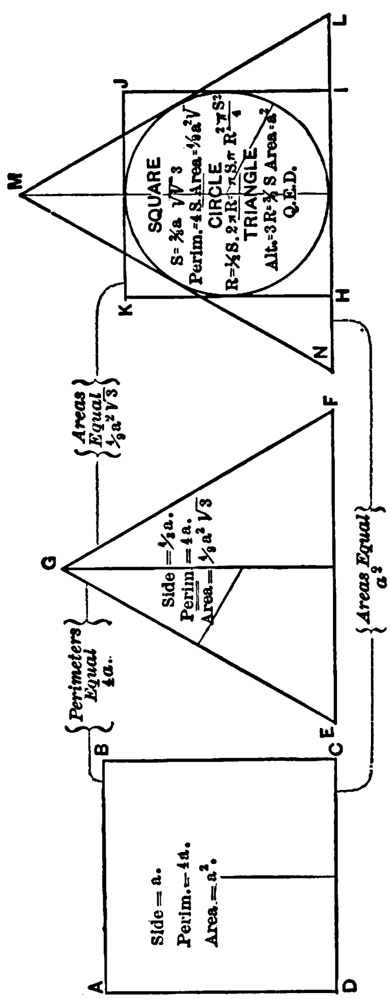
And now for its application. The question proposed was, to construct an equilateral triangle of equal *area* to a *given* square.

"THE UNIVERSAL PROBLEM,"—SPECIAL CASE.

Let $ABCD$ be a given square whose side is a, and whose area is $\bar{a}^2$; then, by the universal problem, will the equilateral triangle LMN, whose side is $\frac{2a}{\sqrt{\sqrt{3}}}$, have the same area.

Following the enunciation just given, the perimeter of the triangle EFG was made equal to $4a$. Each side is thus $\frac{4}{3}a$ in length. The area of this equilateral triangle was calculated and found to be equal to $\frac{4}{9}a^2\sqrt{3}$. Now, the square root of *this* quantity, or $\frac{2}{3}a\sqrt{\sqrt{3}}$, is, of course, the side of a square, $HIJK$, of equal area. One-half ($\frac{1}{3}a\sqrt{\sqrt{3}}$) of this side is the radius PO of the inscribed circle. Three times this radius, or $a\sqrt{\sqrt{3}}$, is the altitude of a circumscribed equilateral triangle whose area is consequently $\bar{a}^2$, or that of the given and original square.

But the principle we have thus employed is even more general yet. We have thus far applied it only to *regular* figures: it applies to irregular figures equally as well. It is a universal principle, by means of which a polygon of any form, regular or irregular, may be constructed, such that its area shall be equal to that of any other given (regular or irregular) polygon.



THE UNIVERSAL PROBLEM AND ITS PYRAMIDAL SOLUTION.

Fig. 47.

The "universal problem" may then be enunciated as follows:—

If any polygon P' (let it be regular or irregular) of any number, n', of sides, be constructed so that its perimeter shall be equal to that of a given polygon P (regular or irregular), and of any number, n, of sides, and a third polygon P'' be constructed similar to P, but of the same area as P', then the area of a fourth polygon P''', similar to P', and similarly circumstanced to the circle about which P'' is drawn (i.e., of equal apothegms), will be equal to the area of the original polygon P.

Let P be any *given* polygon (regular or irregular), and let n equal the number of elementary triangular segments into which it may be divided. Denote by a and S respectively the *mean* lengths of the altitudes and bases of such segments. Call nS its "ruling perimeter," then will $nS\,\frac{a}{2}$ equal its area.

Let it be required to put this area $nS\frac{a}{2}$ into the form of any other polygon P' (regular or irregular) of n' elementary triangular segments.

By the principles of similar geometric figures, construct P' so that its ruling perimeter $n'S'$ shall be equal to nS, and determine all its elements. Let the resulting mean altitude of its triangular segments be a'; then will its area $=\left(S'n'\frac{a'}{2}\right)=Sn\frac{a'}{2}$.

Construct now a third polygon P'', *similar* to the original one (P), but of *area* equal to that of P'; i.e., $=Sn\frac{a'}{2}$.

Then, since P and P'' are similar, we have

$$Sn\frac{a}{2} : Sn\frac{a'}{2} : : a^2 : (a'')^2;$$

hence

$$(a'')^2 Sn\frac{a}{2} = a^2 Sn\frac{a'}{2},$$

and

$$(a'')^2 = \frac{a^2 Sna'}{Sna} = aa',$$

or

$$a'' = \sqrt{aa'};$$

therefore,

$$\left(Sn\frac{a'}{2}\right) \div \sqrt{aa'} = nS'' = \text{the ruling perimeter, etc.}$$

Thus all the parts of P'' may be constructed.

Finally, construct P''' *similar* to P' (i.e., of the required form), but whose mean apothegm shall equal that of P''; i.e., $a'''=a''=\sqrt{aa'}$. By similarity of polygons determine all its parts.

Then

$$\text{Area } P' : \text{area } P''' : : (\text{apothegm } P')^2 : (\text{apothegm } P''')^2;$$

i.e.,

$$\frac{Sna'}{2} : \text{area } P''' : : (a')^2 : (\sqrt{aa'})^2,$$

or

$$(a')^2(\text{area } P''') = \frac{Sna'}{2}(\sqrt{aa'})^2.$$

Therefore,

$$\text{Area } P''' = \frac{Sna'(\sqrt{aa'})^2}{2(a')^2} = Sn\frac{a}{2},$$

which equals the originally *given* area of P. Q.E.D.

Hence the universal problem is demonstrated.

The pyramid diagram (i.e., the "architect's ideal" form thereof) is a geometric representation of that special case of the universal problem which concerns circles and squares.

Its theorem may be enunciated as follows: If a circle be described whose circumference is equal to the *perimeter* of a given square, and the area of such circle be put into the form of a second square, then the area of a second circle inscribed in this second square will be equal to that of the original square.

Proof (*algebraic*). — Let $2x =$ the side of any square, R the radius of the equal-perimeter circle, and R' that of the equal-area circle.

Then $8x =$ perimeter of given square, $4x^2 =$ its area, and $x2\sqrt{2} =$ its diagonal, etc.

By construction, it is required that $2\pi R = 8x$; and it is to be proved, that, by following the method laid down in the enunciation, $\pi(R')^2$ shall equal $4x^2$.

Since $2\pi R = 8x$, $\quad R = \frac{4x}{\pi}$.

Area of first circle $\therefore = \pi R^2 = \pi\left(\frac{4x}{\pi}\right)^2 = \frac{16x^2}{\pi}$.

Hence the side of an equivalent square $= \sqrt{\frac{16x^2}{\pi}} = \frac{4x}{\sqrt{\pi}} = 4x\frac{1}{\sqrt{\pi}}$.

Now, the radius, R', of a circle that shall exactly inscribe the square whose side is $4x\frac{1}{\sqrt{\pi}}$, is, of course, equal to $\frac{1}{2}$ such side; i.e., $= 2x\frac{1}{\sqrt{\pi}}$. But if

$$R' = 2x\frac{1}{\sqrt{\pi}},$$

then

$$\pi(R')^2 = \pi\left(2x\frac{1}{\sqrt{\pi}}\right)^2 = \pi\frac{4x^2}{\pi} = 4x^2. \qquad \text{Q.E.D.}$$

Before turning from the algebraic consideration of the pyramid diagram, it will be as well to demonstrate the equality of area which exists between the pyramid triangle, the circle upon its altitude, and the square upon the radius (R') of the circle of equal area to the pyramid square.

(1) We have already seen that $R =$ to altitude of pyramid triangle $= \frac{4x}{\pi}$, and that its base $= 2x$; hence the area of this triangle $= \frac{1}{2}(\text{alt.})(\text{base}) = \frac{1}{2}\left(\frac{4x}{\pi}\right)(2x) = \frac{4x^2}{\pi}$.

(2) But if R (the altitude of this triangle) is equal to $\frac{4x}{\pi}$, $\frac{1}{2}R = \frac{2x}{\pi}$ is the radius of a circle upon it as a diameter. The area of this circle $= \pi\left(\frac{2x}{\pi}\right)^2 = \frac{4x^2}{\pi}$.

(3) Since, also, R', the radius of the equal-area circle, is $\frac{2x}{\sqrt{\pi}}$, its square (i.e., a square constructed upon it as a side) $= \left(\frac{2x}{\sqrt{\pi}}\right)^2 = \frac{4x^2}{\pi}$. But these three areas are all equal. Q.E.D.

The circumference of the circle on the altitude of this triangle as a diameter $= 4x$. Other interesting relations in this same series of figures (see diagram of "boss") are as follows:—

$$R' = \frac{R}{2}\sqrt{\pi};$$

$$R = \tfrac{2}{5}(R')^2;$$

$$R - R' = OG = R(1 - \tfrac{1}{2}\sqrt{\pi}).$$

Denote the distance GP by y. Then

$$y = R' - (R - R') = 2R' - R$$

$$\therefore\ y = R(\sqrt{\pi} - 1).$$

Denote by $x - y = z$, the shortest distance from centre (N) of square $ABDE$ to the circumference (P) of circle of equal area.

$$x - y = z = \frac{R}{4}(\sqrt{\pi} - 2)^2.$$

In the case of the boss, these several values are numerically as follows: —

$$2x = 5;$$

$$R = 3.1830988618232243060 2500 +;$$

$$\pi R = 10. \quad 2\pi R = 20;$$

$$\pi R^2 = 31.830988, \text{ etc.};$$

$$2R = D = 6.3661977 +;$$

$$\frac{R}{20} = \frac{1}{2\pi} = .15915494, \text{ etc.};$$

$$R' = 2.82094791775 +, \text{ etc.}, = \frac{5}{\sqrt{\pi}};$$

$$\pi(R')^2 = 25;$$

$$(R')^2 = 7.957747715 +; \; 2\pi R' = D'\pi = 17.7245385091 +;$$

$$2R' = D' = 5.64189583550 +; \; (D')^2 = 31.830988618232 +;$$

$$\frac{D'}{10} = .564189583550 + = \text{radius of circle whose area} = 1 = \frac{R'}{5}.$$

$$\text{Area of } \triangle = Rx = 7.9577747 +; \; y = 2.45879697350 +, \text{ etc.};$$

$$z = x - y = .04120302650, \text{ etc.}, = \text{distance } PN.$$

There are converse and collateral problems to the above which I cannot even touch upon: in fact, I must close the study of the whole topic. The half has not been told. Though I have devoted two hundred pages to this interesting subject, I could as easily devote two thousand; but, should I do so, even then the topic would have been barely touched upon: we would have the same unsatisfactory feeling at its close, and be oppressed with a sense almost akin to that of mental suffocation. Personally, I am dissatisfied — almost despairingly so — at what is now known of this greatest wonder of the world. Into its mysterious shadows I see, vanishing, threads that apparently lead through the labyrinths of every branch of human knowledge.

The pyramid is indeed a mystery. The Promethean idea so thoroughly pervades its plan and realization, that, by it, high science, accurate history, and perfect art, are blended in most exquisite harmony.

In it, *Art* is taught by the beauty, grandeur, endurance, and perfect fitness of the monument itself; *Science* by its universal earth reference and commensurability; *History* by its symbolic architecture; *Religion* by the story and purposes of its erection, and by the marvellous prophecies and teachings built in stony parables into its everlasting walls; *Astronomy* by master pointings to the stars and heavenly cycles; *Astrology* by the true readings of the constellations, and their order of precession; *Chronology* by time references, tubes, and passages, and an index that marks unerringly the initial year of the *annus magnus; Natural Geography* by its unique location at the gate of universal commerce and the centre of the inhabited earth; *Physical Geography* by its thorough knowledge and appreciation of the bearings of temperature, pressure, and climate upon normal human life; *Agriculture* by its very name, and by the placing in its safest chamber of a just measure of the staff of earthly and the symbol of eternal life; *Mineralogy* and *Geology* by the wisdom displayed in the selection of a material that should not only endure, but secure, a certain constant specific heat and gravity; *Metrology* by the irresistible deductions from facts built thus in the form of all its elementary units; and, finally, *Man* himself by its practical co-ordination of all the wants of his best development as a civilized and social being.

APPENDIX B.

RATIONAL CIRCULAR SUBDIVISION.

"And round about the throne were four and twenty seats: and upon the seats I saw four and twenty elders sitting." — REVELATION iv. 4.

"And under the brim of it round about there were knops compassing it, ten in a cubit, compassing the sea round about." — 1 KINGS vii. 24.

RATIONAL CIRCULAR SUBDIVISION.

"On this subject he (our ancient brother mason, Pythagoras) drew out many problems and theorems; and, among the most distinguished, he erected this, which, in the joy of his heart, he called *Eureka*, in the Grecian language signifying, *I have found it*. . . . It teaches Masons . . ." — TRESTLE-BOARD.

THE following paper upon the "Division of the Circle," written by Mr. Jacob M. Clark, C.E., at the request of the author,[1] is appended because of its importance. Universal metrology cannot avoid the discussion of the *circle;* and it is a matter of supreme importance to the *practical* man, what mode of treatment this primary geometrical form shall receive. Babylon has long advocated, and from her we have accepted, the division into 360 parts of 60 degrees each. Metric France attempted to make the subdivision into 400 parts, and thence decimally downward, but failed. The Great Pyramid advocates, as the truly rational division, one into 240 parts, and thence decimally or otherwise downward. The latter will undoubtedly prevail, and the day of its adoption may now be regarded as *close at hand;* for its notably neat relation to the standard-time division of the watch, and to the requirements of geography and astronomy, insure its claims.

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[1] NEW YORK, Dec. 22, 1883.

MY DEAR LIEUT. TOTTEN, —

I have written out pretty fully my argument on the Division of the Circle. If in any respect it will be of use to you, I shall be pleased. I know it may be found defective in logical arrangement; for, you know, I have to write a sentence at one time, and another at another, under different surroundings. You may, therefore, find it "candid, discursive, and didactic."

I am fully aware that habit, antiquity of usage, prejudices of education, politico-economical influence, and the like, will be generally against the suggestion. They are the same "thrones, principalities, and powers," that are always invoked to bolster up that old serpent, the Devil, and original sin. . . .

Truly yours, JACOB M. CLARK.

THE DIVISION OF THE CIRCLE.

From the nature of the problem, the argument must be in part tentative and inductive, rather than synthetic or analytical.

Hitherto, in the treatment of the circle for primary arcs, geometers seem to have stopped at the $\frac{1}{15}$. Below this they have for many centuries divided the circle by 24 for the diurnal cycle, and for general purposes, somewhat arbitrarily, by 36, with continued sexagesimal subdivision in each case.

In strict geometry, so far as known, the only general method of subdividing circular arcs is by simple bisection. There are, however, a limited number of commensurable arcs which can be found by other methods.

Taking radius, or some aliquot part of it, as a function, we easily obtain the $\frac{1}{5}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{12}$.

By means of extreme and mean ratio applied to radius, we have, by a very simple process, the $\frac{1}{5}$ and $\frac{1}{10}$.

It is by *comparison* of the $\frac{1}{6}$ with the $\frac{1}{10}$ that we obtain the $\frac{1}{15}$.

Now, while comparison is not a *general* method of subdivision in circular arcs, for the purpose of obtaining a common measure, it is susceptible of being carried somewhat farther. To understand this more fully, let us apply the *general* method of bisection to the arcs already found. The result is the following fourfold series: —

A. 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ . . . .
B. $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{24}$, $\frac{1}{48}$ . . . .
C. $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{40}$, $\frac{1}{80}$ . . . .
D. $\frac{1}{15}$, $\frac{1}{30}$, $\frac{1}{60}$, $\frac{1}{120}$, $\frac{1}{240}$ . . . .

It is observable in the above group, that the smallest commensurable arcs which have commensurable functions are the $\frac{1}{8}$ and the $\frac{1}{12}$. Compare them, and their greatest common divisor is the $\frac{1}{24}$. From the distinctive peculiarity expressed, it may be regarded as the major unit of circular measure, unless reasons can be afterwards found for displacing it from that position. We already see that the $\frac{1}{24}$ can be obtained without bisection Let us see how far we can carry this principle of comparison without resorting to bisection at all.

By comparing the $\frac{1}{4}$ with the $\frac{1}{5}$ we obtain the $\frac{1}{20}$.
By comparing the $\frac{1}{12}$ with the $\frac{1}{20}$ we obtain the $\frac{1}{30}$.
By comparing the $\frac{1}{8}$ with the $\frac{1}{10}$ we obtain the $\frac{1}{40}$.

$$\left.\begin{array}{l}\text{By comparing the } \frac{1}{12} \text{ with the } \frac{1}{15}\\ \text{Or by comparing the } \frac{1}{10} \text{ with the } \frac{1}{12}\\ \text{Or by comparing the } \frac{1}{24} \text{ with the } \frac{1}{40}\end{array}\right\} \text{we obtain the } \frac{1}{60}.$$

$$\left.\begin{array}{l}\text{By comparing the } \frac{1}{24} \text{ with the } \frac{1}{30}\\ \text{Or by comparing the } \frac{1}{30} \text{ with the } \frac{1}{40}\\ \text{Or by comparing the } \frac{1}{20} \text{ with the } \frac{1}{24}\\ \text{Or by comparing the } \frac{1}{40} \text{ with the } \frac{1}{60}\end{array}\right\} \text{we obtain the } \frac{1}{120}.$$

It is apparent that we can proceed no farther without resorting to bisection somewhere. And it is evident, too, that the $\frac{1}{120}$ is the greatest common divisor of all the arcs in the first four columns, — i.e., of all the commensurable arcs which have commensurable functions, — and of those which are obtainable by means of extreme and mean ratio, and of those which result from comparison of these arcs among themselves, *taken together.*

Now, if we bisect all the arcs in the fourth column, it is apparent that the $\frac{1}{240}$ is the greatest common divisor, not only of all those in the fifth column, but of all in the other four. And the same arc, the $\frac{1}{240}$, is also obtainable by bisecting some other members of the fourth column, and then instituting again the principle of comparison. That is, —

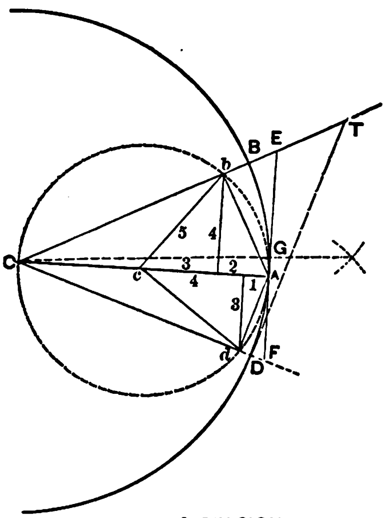
$$\frac{1}{16} - \frac{1}{24} = \frac{1}{48}.$$
$$\frac{1}{16} - \frac{1}{20} = \frac{1}{80}.$$
$$\left.\begin{array}{r}\frac{1}{48} - \frac{1}{40}\\ \text{Or } \frac{1}{80} - \frac{1}{120}\\ \text{Or } \frac{1}{60} - \frac{1}{80}\\ \text{Or } \frac{1}{15} - \frac{1}{16}\end{array}\right\} = \frac{1}{240}.$$

So that, after all, it is not strictly a matter of indifference where we resort to bisection, if, after having proceeded as far as the $\frac{1}{120}$, we desire to secure the most elementary result; i.e., to exclude unnecessary repetition of methods. We see that we need not resort to bisection directly more than once in order to secure the desired result, so far as it can appear in the fifth column.

And, in this search for the purest division, it has seemed to me a postulate, that all known methods should be exhausted, but that, in order to secure the greatest pure common divisor of the results, no method should be repeated, unless it be that of making a final comparison, after all the methods shall have been once applied at the most primitive or fundamental point. Otherwise the result would be vitiated by unequal weight among the methods.

Accordingly we are led to examine whether there is not some one of these arcs in the fourth column which has some peculiarity not possessed by the others, and which connects with all the methods, and which will therefore indicate precisely where the method of bisection ought, firstly and finally, to be applied.

Now, the octant is precisely such an arc. It is the largest arc in the fourth column. Moreover, we have just seen, that, by bisecting it, we can obtain by comparison all the other members of the fifth column, and among the rest the $\frac{1}{240}$ in several ways.



THE 240° DIVISION
Fig. 48.

It is also the only known commensurable arc with a commensurable function, which is at the same time the sum of two incommensurable arcs, each of which has a commensurable function. Its tangent is equal to radius; and it is composed of two incommensurable arcs, whose tangents are respectively equal to $\frac{1}{2}$ and $\frac{1}{3}$ radius. This latter property, well known to analysts, may, without resorting to the rules of trigonometry, be shown by a strictly geometrical test, which, while it reveals the wonderful power of the Pythagorean form of the right-angle triangle, 3-4-5, at the same time discloses a direct connection of the octant with all the methods, such as cannot be found in any other known arc.

In the figure, let the radius CA of the main circle equal 10. Bisect this radius in c, and with $Cc = Ac = 5$ as a radius describe the minor circle $CbAd$. Upon opposite sides of this minor diameter apply the 3-4-5 triangle in reverted positions, the acute angles resting in c. The complementary angles will be in the minor circumference at b and d respectively, and the angle bcd will be a right angle. Draw the minor chords bA and dA; also Cb and Cd, produced indefinitely, so as to intersect the greater circle in B and D respectively. From D draw a tangent, intersecting CB produced in T. Draw also through A a tangent, intersecting Cb and Cd produced in E and F. The arc BAD is an octant, whose tangent DT is known to be equal the radius; and, by simple triangles, the tangents AE and AF of its component arcs are equal respectively to $\frac{1}{2}$ CA and $\frac{1}{3}$ CA. The angle ECA is the one by means of which we arrived at extreme and mean ratio, and obtained the chords of the $\frac{1}{5}$ and $\frac{1}{10}$.

The tangent of the octant BCD is equal to the chord of the $\frac{1}{6}$. Half of it, the tangent of BCA, is equal to the cosine of the $\frac{1}{6}$ and the sine of the $\frac{1}{12}$; taken as the height of a segment, it gives the chord of the $\frac{1}{3}$. If the height of a triangle equals the radius, it is the factor by which we multiply the base to find the area, and, indeed, the circumference to find the area of a circle. One-third of it, the tangent of ACD, is the factor by which we multiply the area of the base of a cone or pyramid (the height being equal to the radius), or the surface of a sphere, to obtain the solidity.

Thus far these properties of the octant have been examined in the light of simple geometry. It may now be added, that analysts have found (see Calculus), that, by a simple but slowly converging series founded on the tangent, we can compute the length of the arc, and so of the circumference; and that two analogous but rapidly converging series founded on the tangents of its component parts, furnish the best and shortest known means of calculating the length of the circumference to any required degree of exactness.

The octant, then, though not a *unit* of circular measure in the strictest and most exhaustive sense, is a *dominant* arc. Being thus related to all the other arcs, and to the methods by which they are obtained, and being itself founded on the perfect application of the square to the circle, it furnishes the final and crucial test. And, from the foregoing considerations, it is claimed, that as a matter of pure geometry, and in every aspect of mathematical propriety, we should bisect the octant in preference to any other arc.

Bisect it (see figure) in G'. BG, being equal to DG, is then the $\frac{1}{16}$. Compare this with the $\frac{1}{15}$, which, as we have seen, exhausts, on the principle of non-repetition, all the methods except bisection, and we obtain the $\frac{1}{240}$, the greatest common divisor of all the *geometric* arcs down to that limit.

And we see also, that this division by 24 emphasizes the geometry of the round bodies in a manner which no other grand division of the circle can possibly do. From the cylinder on the diameter down to the cone erected upon radius, all the relations are expressed in terms of integer hour-arcs.

We have, then, —

First, As a grand unit of circular measure, the $\frac{1}{24}$, or hour-arc, the *greatest common divisor of all the commensurable arcs which have commensurable functions.*

Second, As to the method of subdivision, we have, for the *geometric degree*, the *greatest common divisor*, so to speak, *of methods and commensurable functions.*

And, between the two, the modulus is the product of the only two geometric divisors which subsist; i.e., $2 \times 5 = 10$. And, between those limits, that modulus, and that alone, covers all the geometric arcs. Below the $\frac{1}{240}$, so far as we know, there is but one general method, bisection. And for finer divisions we must resort to some empirical method, unless we choose to continue the binary arithmetic indefinitely. But the general modulus of arithmetic being 10, it is difficult to divine a reason why the decimal method should not prevail indefinitely downward from the $\frac{1}{240}$.

Scholium. — This decimalization accords with the *general* practice of the ancients, so far as we can judge from their itineraries. Having fixed the major unit, they subdivided decimally. The Egyptians so subdivided from the $\frac{1}{8}$, the Persians from the $\frac{1}{6}$, the Syrians from the $\frac{1}{16}$, and the ancient Hebrews from the $\frac{1}{18}$. Eratosthenes seems to have undertaken it from the $\frac{1}{25}$, and Posidonius from the $\frac{1}{24}$. The French have latterly attempted the same thing from the quadrant.

We may here notice that the number 3, as a circular divisor, disappears below the $\frac{1}{24}$. It is peculiar to circular measure, but only above that limit. Below that, it can only be introduced empirically. There is no reason why it should be introduced into the general modulus of arithmetic at all. And the prime division of the circle by 36, or by 18, is as ungeometrical as that by 25. And the sexagesimal principle of subdivision is doubly so. The other grand divisions spoken of are more rational, but imperfect because not exhaustive.

Precisely when the present division of the circle into 360°, and the sexagesimal principle of subdivision, were invented, or whether the two were coeval, is not certainly known. They are traceable to Babylon, and generally assigned to the first, or Nimrodic, monarchy. The whole structure, like its cousin the duodecimal arithmetic, is the work of mathematical cranks, who have attached undue significance to the multiples and powers of the number 3. The systems just referred to, — the Hebrew in its decimalization below the grand unit (which was double of the one in this system), — and the others throughout, are strong historical protests against it. They have failed, from political causes in the main, but partly through imperfections in the grand division. An exception in the latter respect appears in the conclusion of Posidonius (about 90 B.C.), who, whether by accident or design, seems to have struck the true geometric division. The combined effect of causes, however, which have contributed to the prevalence of the present system, belong to the historian rather than to the mathematician.

But, in view of the present wide-spread discussion on the subject of simplifying the correlation of our own weights and measures, the question is put to thinkers plainly, and with some confidence, whether, in this matter of circular division, the gradual displacement system by one founded on the strictest demands of simple geometry, will not, in the end, as a matter of general economy, by simplifying relations, and reducing the aggregate labor of operators and calculators in all departments of applied mathematics, be worth a thousand times more than its cost.

JACOB M. CLARK.

NEW YORK, Dec. 22, 1883.

APPENDIX C.

METROLOGICAL NOTES AND QUERIES.

"Who hath measured the waters in the hollow of his hand, and meted out heaven with the span, and comprehended the dust of the earth in a measure, and weighed the mountains in scales, and the hills in a balance?" — ISAIAH xl. 12.

METROLOGICAL NOTES AND QUERIES.

"The sands and their numbers I know,
And the measure I know of the ocean."

DELPHIC ORACLE.

[NOTE I.]

THE ORIGIN OF THE GRAIN.

HASWELL places the average weight of a bushel of wheat at 60 avoirdupois pounds. Hence, as there were 32 bushels in the ancient coffer (and Anglo-Saxon chaldron), its weight of wheat was 1,920 avoirdupois pounds. It will be noticed that 10,000 times this number, 19,200,000, is the weight of our rectified coffer in "grains." But the involved relation is even more remarkable when we count the kernels of wheat thus weighed. Now, there are, upon an average, 17,812,500 ± kernels of "ripe wheat taken from the middle of the ear" in a cofferful.[1]

As the capacity, therefore, of the coffer, is 71,250 cubic inches, each cubic inch of space will hold exactly 250 kernels of ripe wheat. It will be noticed that this quantity is remarkably close to the number of weight grains (252.6937) now taken as the value of a cubic inch of pure water at its maximum density (39.8° Fah., 30″ bar., Haswell); so that we cannot refrain from conjecturing that this fact may have had at least a remote bearing upon the original selection of the particular number of elementary Anglo-Saxon weights in a cubic inch of the natural weight material, water. It is, therefore, probable, that at standard circumstances (ther. 68° Fah., bar. 30″, and hygro. $\frac{1}{5}$ wet) the average number of wheat-kernels which could be contained in the volume occupied by a pyramid cubic inch of water would not materially differ from 250, and the division of this amount

[1] Haswell estimates that a bushel contains 556,290 kernels; hence, even at this estimate, 32 such bushels would contain 17,801,280.

of water into the same number of elements would thus arithmographically realize the actual bread capacity (i.e., in wheat-kernels) of the same cubic volume.

To many, of course, the unlooked-for character of these results, and their surprising fitness, stamp them at once with such an air of mystery, that they will be disposed to reject them as suspicious, and as merely the products of so-called "trick-arithmetic." But to others they will bear upon their face a stamp of too much genuineness to be set aside at the mere objection of the doubter, who ignores the very records of the subject under discussion. We know, for instance, that it was in just such a way that our own, not so very remote, forefathers actually arrived at their three-barley-corn inch. So, too, by the entirely independent regulations of ancient China, an inch was by statute made equal to the average breadth of 10 millet seeds, and that of 100 millet seeds, or 10 inches, measured and established the standard foot. The very next step in such a system must have been to obtain the average number of such elements in a superficial area, and the next the number in a given and handy cubic volume. Such steps would have been, in fact, the only legitimate and logical ones in the passage from linear measure to that of area, and thence to that of simple capacity. So, likewise, in passing from capacity to weight measures, it would have been, and was, most natural to observe the *arithmetical* relation for some time, even after another substance — water — had been found to be the most fitting basis for the latter. In some such way alone could the numerical sequence and *rationale* have been preserved; and that it was so preserved, there is ample testimony at hand in the history of metrology.

Grain measure was the end and object, as its name implies, of the pyramid (pyros-metron); and the founding of an elementary weight upon the arithmetical value of the volumetric capacity for wheat-kernels would, in view of the beauty of the resulting subdivision, have been neither unwise nor impracticable. But, be this as it may, the fact of their remarkably close approximation remains, and, as we have seen farther back, gains credence from entirely independent sources.

The objection, that our results are only those of "trick-arithmetic," is of no moment; for such an objection holds equally good against any *system*. It would sweep away our whole decimal system, and brand as unworthy any employment of the properties of the number 9 in mathematical work. The term, "trick-arithmetic," is, in fact, short-sighted, and has been invented by some very poor mathematician to cover his ignorance of the properties of numbers; for it does not exist in the sense intended.

But, before closing this subject, let us examine its numerical bearings a little further. Since there are 1.78125 cubic inches of volume in the ounce of capacity, and since each such cubic inch holds on an average 250 kernels of ripe wheat, the number of such kernels in an ounce of capacity is 445.3125: 10,000 times this is therefore the number contained in a "quarter." There being 16 ounces in the pint, it will contain 7,125 wheat-kernels, a number which is one-tenth the cubic capacity of the chaldron itself in inches. Now, one-half of 445.3125 is 222.65625, and represents the number of wheat-kernels in half a capacity ounce. The latter number, however, is one-tenth the volume in cubic inches of the thirty-second of the chaldron, which is the bushel; that is, the bushel contained 2,226.5625 cubic inches, and 556,640.625 kernels of wheat. But as the bushel weighs only 60 pounds, or the coffer of wheat weighs but 1,920 pounds, it would have been balanced by $1{,}920 \times 28.5$ cubic inches of water $= 54{,}720$ cubic inches. Hence, by weight, a bushel of wheat was balanced by 1,710 cubic inches of water. The question now arises, whether, at any time in the past history of Saxon metrology, such a method of measuring a bushel of wheat — namely, by balancing it against what was very nearly a cubic foot of water ($\sqrt{1710} = 11.96''$) — could have obtained? The answer is, that it would have been a most natural method, and one — measuring capacity by weight — still very common in the commerce of cereals, etc. The next question which presents itself, is, whether the measure intended to hold the water might not, either through the ignorance or cupidity of an illiterate age, have come at length to be employed as the actual measure of the wheat itself? Such a misappreciation of the original system may easily have come about, and its assumption may perhaps shed some light upon the origin of the very short bushel of the fifteenth century.

The system demands that a thirty-second of a coffer, or the capacity of 2,226.5625 cubic inches of wheat, shall be balanced by 1,710 cubic inches of water, and that each volume shall weigh exactly 60 pounds. On the other hand, a thirty-second of the water weight of the coffer would give $78\frac{1}{8}$ pounds, while 1,710 cubic inches of wheat would have weighed but $46\frac{1}{12}$ pounds.

It is a noticeable fact, that, in the reign of Henry VII. (A.D. 1496), — one of the most archaic periods in the history of British metrology, — two just such bushels were in actual use. The one — a bushel which the people at length refused to tolerate — was of but 1,792 cubic inches capacity, and yielded a coffer of only 57,344 cubic inches. The other was a wonderfully close approximation to the ancient standard, and held 2,224 cubic inches, giving a coffer of 71,168 cubic inches, or one only $\frac{1}{869}$ short of the true standard. In the very next reign, the gallon was fixed at 282 cubic inches (very close to the rectified value, 285), the bushel at 2,256, and the chaldron therefore at 72,192 cubic inches.

Now, in all of these approximations and returns to values which are pointed out as probably related to those of the ancient pyramid standards, there is certainly too much system to be attributed solely to blind chance and accident. President Barnard considers that it "argues lunacy" in the man who makes the statement that the pyramid coffer has been the standard from which the corn measures of Great Britain have been derived. It strikes us, rather, that he who persistently resents the natural conclusion that these things are related, and *must* have been somehow related, no matter how remotely, in the minds of the originators of the Anglo-Saxon metrology, violates every rule of logical ratiocination. Of course, these measures have varied more or less during the forty centuries that have intervened since they first "came up out of Egypt." We do not presume to maintain that they have been miraculously preserved always exact, any more than in the face of facts we presume to maintain that the English inch of to-day is absolutely equal to the American inch, and that both are, or either of them is, at their ancient cosmic value. Nevertheless, it may be fairly maintained, that these measures have *not* varied in the long-run "materially." The proportions of an ordinary door are probably about the same the world over, and have probably been maintained at a constant average ever since the human structure attained its normal height. So, too, ever since wheat became the staff of life, and so long as it shall continue to be a staple of national and international commerce, the ratio of its bulk and weight, its specific gravity, in fact, has maintained, and will continue to maintain, its bushel around that handy size which experience finds not only to facilitate its ready manipulation, but which the human mind recognizes as a direct reference to the necessary "tabular numbers" of general metrology which we have inherited. But there are entirely independent considerations which go to establish most conclusively a direct connection, through blood and race inheritance, between the measures of northern Europe and those of ancient Israel and Egypt. Unfortunately, for lack of space, we cannot touch upon these at all in this place; and probably, if we could, they would be classed by Mr. Proctor, President Barnard, and their followers, in the catalogue of "the vagaries of lunatics." Our object is now, more particularly, to establish the eminent superiority of present Anglo-Saxon metrology over all other competitors, and to show how little rectification will be required to render it a system whose prestige can never thereafter be shaken.

The objection may be raised at this point, that 250 is not the number arrived at in our investigations as that expressive of the grains of weight in a cubic inch of water, and whereby it is advised that our present system of metrology be rectified. The latter number is $15360 \div 57 = 269.47 +$, or is $19.47 +$ units too great. But an examination will show that this objection is itself without weight. The 250 grains so closely approximated to by the grain weight now in Anglo-Saxon use, were originally grains of *capacity*, not of water weight, and indicate the average *number* of ripe wheat-kernels from the middle of the ear that a cubic inch of volume will actually hold. We thus perceive that there is an entirely different stand-point from which to consider this question, and one far more appropriate; namely, from that of weight itself. Let us therefore examine the subject from this new stand-point.

A cofferful of wheat-kernels will weigh less than one filled with water in the ratios of their respective specific gravities.

Now, as a cofferful of wheat contained 32 bushels of 60 pounds each, it weighed 1,920 pounds, or balanced $1920 \times 28.5 = 54720$ cubic inches of water. But a cofferful of water contained 71,250 cubic inches. Hence the specific gravities of wheat-kernels and water are as

$$54720 : 71250, \text{ or as } 192 : 250 !$$

From the foregoing it follows, that $\frac{1}{192}$ of the weight of an even cofferful of wheat-kernels will exactly balance $\frac{1}{250}$ of a cofferful of water. This is the weight of 285 cubic inches of water. Now, if a 100,000th of this volume be taken as a terminal quantity, so far as *weight* is concerned, it will be equivalent to .00285 cubic inches of water at standard circumstances, a value already arrived at without reference to any kind of cereal as the true "ultimate" of our rectified system.

But since 192 times the water-filled coffer is equivalent in weight to 250 times that of the same space filled with wheat-kernels, one of the water parts will weigh $\frac{250}{192} = 1.30208\frac{1}{3}$ parts of the terminal (.00285), just determined; i.e., will be equal to .0037109375 (exact) cubic inches, or give us the actual value of our rectified *weight*-grain! And the standard weight of a kernel of ripe wheat taken from the middle of the ear will therefore be .003072 (exact) cubic inches of water, or $1.0\dot{7}78947357\dot{}$ "ultimates," or .82782315520 of a "grain."

It will take 322.265625 (exact) such kernels to balance a cubic inch of water; and, finally, the specific gravity of such a standard kernel will be exactly $.768 = 1536 \div 2$. What the modern determination for this latter quantity — the specific gravity of ripe wheat in the kernel — actually is, I have no means of knowing, as none of the tables within my reach give it; so I leave it for future determination, and as a sort of crucial test of some of these deductions.

It is of no consequence to me in what particular method these facts were arrived at in ancient Egypt. They are matters of ratio and number, and could have been determined by treating, as we have done, any equivalent volumes of wheat, water, and 5.7 density material; i.e., by comparison and by extended series of actual experiments as to weight and number of kernels, or by a study of number in the abstract, cubic volume, and mean density. The facts themselves are not manufactured, but are of cosmic import, and the question a scientific one as to the actual ratios running through nature.

President Barnard ridicules the idea of man having, at that early day, been able to attain by scientific process and discovery to any such accuracy of knowledge as the pyramid theory demands. In this position we are disposed in part to agree with him. From the stand-point occupied by the closely corporated modern school of science, there is no other view to be taken. But the dilemma is only intensified by President Barnard's claim, for mere ridicule and denial will not sweep away such an array of data as are built into the Great Pyramid. Whether determined or not by self-education, and understood or not in the land that the monument overshadowed, they were none the less actually there, if only to be locked up and sealed down in everlasting masonry. And this, after all, is the whole gist of the mystery of this sign and wonder of the land of Ham, — that such knowledge should have been possessed, and not understood, — unless we accept at once the religious theory of the monument, and admit that it may have been, and probably was, made after a model shown in vision, as were the tabernacle, the temple, and the still earlier ark of Noah.

As a last resort, we may imagine some objector claiming that the whole of this system is the mere result of "arguing in a circle." Such an objection, however, though easily made, is a very powerless one, when we consider the great antiquity of the system, its actual use in Anglo-Saxondom from time immemorial, the widely spread traces of it all over the earth, its eminently practical character, its scientific beauty, its cosmic, geometric, and numerical features, and its endless possibilities. So that, even were the objection a valid one, we would be tempted to amend and enlarge it by calling it an argument about the surface of a sphere, and congratulate ourselves that we had found so firm a footing.

To recapitulate, therefore, the facts we have been considering, we subtend the following table : —

250.	= No. of kernels of wheat in 1 cu. in. volume.
322.265625	= No. of kernels of wheat to balance 1 cu. in. of water.
269.47368 +	= No. of *weight* "grains" to balance 1 cu. in. of water.

350.87719+	= No. of *weight* "ultimates" to balance 1 cu. in. of water.
371.09375	= No. of cu. in. of water to balance 100,000 weight grains.
307.2	= No. of cu. in. of water to balance 100,000 kernels wheat.
285.	= No. of cu. in. of water to balance 100,000 weight ult.
400.	= No. of cu. in. occupied by 100,000 kernels wheat.
7125.	= No. of wheat-kernels in a pint measure.
9277.34375	= No. of wheat-kernels in an avoirdupois pound.
.768	= standard specific gravity of ripe wheat.

[NOTE II.]

UNITS AND CONSTANTS.

In due time science will undoubtedly be forced to establish a uniform system of logically correlated units. Chaos itself was order compared with the confusion of terms, ideas, and definitions upon which the present scientific schools base their investigations. Each branch of scholars seems to have evolved its own units with closed doors, and to have jealously excluded all recognition of the fundamental principles of others. Metrology, however, is the universal bond which unites knowledge upon all subjects; and, until its true prerogatives are duly appreciated, the deductions of scholars cannot be more than a mere patchwork of disconnected results. Harmony of fundamental principles is the substratum underlying all the various forms of Nature's manifestation, but discord is now written across the tablet upon which the modern schools have rudely scribbled their uncertain "constants."

The result is, that in order for one school to "tie on" to the facts of another, often involves the translation of an almost inexpressible function. Could liberal representatives from the colleges of every branch of knowledge but unite in a convention upon "Terms and Units," the elevated site might soon be cleared away upon which to erect a monument as stable as that of ancient Egypt; but until there is some such unity of purpose, governed by the logic of harmony, we cannot expect our fabric to be more than one of clay and slime upon a sandy plain.

Now, it seems to the writer that no definition of a unit should involve terms, which, in the several lines of knowledge involved, are not also units; i.e., of the same "degree." For instance, pounds and feet are, in their order, of an entirely different degree from seconds of time. The latter is a real unit. The former are collections of units, and therefore, logically, can no more come together in the definition of a compound unit than bushels and dollars can be logically compared with each other on the same sides of a proportion. A minute in time corresponds to the foot of distance, and the pound of weight, just as inches, ounces, and seconds correspond; and until "unity," as a fundamental idea, means numerically 1, we can have no proper starting-point for universal metrology.

We repeat, therefore, what seems to be a self-evident demand of the logical processes of the mind, that the definition of a compound unit should accept no terms which are not respectively the well-known units in their several and individual lines of study. In the investigations of science conducted upon such foundations, there could be no misconceptions; and, wherever the wheels of one branch come into contact with those of another, we should have a true tangential contact, and not a mutual interference, caused by the cutting of the one into the other.

For instance, let us try and define a compound unit, that of "the RATE of *work*," in a logical manner. This is a proper subject of measurement, and must be measured by *units* if we aim at fundamental facts, and if we wish our definition to be directly available, as unity, in all of the branches involved. In general terms, therefore, we should define the "unit of the rate of work" as the amount of force necessary to raise a unit of weight to the height of one linear unit in one unit of time. If we substitute in the above the actual elementary units, and remain logical, we cannot avoid defining it as, the amount of force necessary to raise one ounce, one inch, in one second. If we confuse the degrees of the things involved, our result has at once an entirely different meaning. Thus the common definition, "the quantity of force necessary to raise one pound one foot high in one second," is in reality a *standard* of the rate of work, and not its unit; since the resistance overcome, and the distance passed over, are the common standards of weight and length, and not at all their units. In terms of the true units, this criticised definition is therefore as follows: The quantity of force necessary to raise sixteen units of weight to a height of twelve units of distance, in one unit of time — and the objections to its fundamental character, as a "unit," become instantly apparent. It will be replied by some, that, after all, this is merely a captious misappreciation of terms, and that the pound and foot are truly scientific units. If this is so, then a minute instead of a second should be taken as the unit of time; for it is of the same degree. But, so far as time is concerned in the working of human mechanisms, the minute would be utterly impracticable. The velocity of even the longest ranged projectile could not be appreciated upon such a basis, and the legitimate unit of time — a second — is naturally retained from sheer necessity.

And here it should be premised, that, as the structure of a series of units upon such principles will involve no ideas save those of a primary character, no further reductions can ever after be necessary, or even possible; while, from them, *standards* of any degree of magnitude and of any degree of complexity that special circumstances may seem to warrant, can instantly be formed. Thus, for instance, suppose the rate of work of a machine is found to be 691,200 units; then, from the very nature of our unit, we may at once say, that the rate of the machine is also 6,912 *standard* foot-pounds, or 3,600 *common* foot-pounds, or that its rate is $1\frac{4}{5}$ common foot-tons, or that it will accomplish 108 foot-tons per minute, or 6,480 per hour, or do the work of $4\frac{196}{275}$ horses per day, etc.

We do not propose to attempt here the definition of all the units of science upon the basis discussed. It clearly involves the united interest and study of liberal specialists to accomplish such a task. But leaving out of present consideration those of electricity, magnetism, sound, light, and of many other branches whose constants are even yet very little understood, we will venture to define a few of the more common units from this stand-point: —

Unit of velocity equals a uniform motion equal to one linear unit per unit of time (1 second).

Unit of force equals that force, which, uniformly accelerating a unit of weight for a unit of time, would produce a unit of velocity.

Unit of work equals the amount of force necessary to raise the unit of weight through the height of one linear unit.

Unit of rate of work equals the amount of force necessary to perform a unit of work uniformly in a unit of time.

Unit of temperature equals the amount of heat necessary to raise the thermometric column one standard degree at standard circumstances.

Unit of heat equals the quantity of heat necessary to raise a unit weight of water through a unit of temperature.

Unit of circumference equals one degree of a circle.[1]

Unit of angular motion equals a uniform motion about a centre equal to one unit of circumference per unit of time.

[NOTE III.]

THE DIMENSIONS OF THE COFFER.

The labors of Mr. W. M. Flinders Petrie, detailed at length in a handsome octavo of 250 pages, comprise the latest work that has been carried on at Gizeh with a view of obtaining both the present measures of the Great Pyramid, and some idea of the *rationale* upon which its proportions were laid out.

The book, unfortunately, is written in a most pointed animus against the metrological, or so-called religious, theory of John Taylor and Professor Smyth, and, as such, has been gladly published almost entirely at the expense of the British Association. So far, however, from accomplishing its ends, the volume has been proved by many able reviews to establish beyond peradventure the soundness of John Taylor's original propositions.

We do not intend to review this work ourselves; since we take it for granted that all who are disposed to study the significance of the proportions of the Great Pyramid fairly, and upon their own merits, and who have Mr. Petrie's book at hand, will also post themselves upon the able replies which the deductions contained in this volume have already received at the hands of writers in "The International Standard" (see volumes for November, 1883, January and March, 1884). Mr. Petrie's measurements themselves, however, are generally excellent, and will always occupy a dominant position in future studies of the monument. Compared one by one with those of Professor Smyth, they

[1] See Appendix B.

will be found to completely vindicate the entire honesty of the latter, and to establish more firmly than ever the footing of the metrological school. Indeed, it is not a little strange, that arriving, by an extended series of measurements, at almost the identical figures of Professor Smyth, and profiting by all the study of their proportions and ratios which had gone before him, Mr. Petrie should have fallen into such serious error as to his deductions, and have allowed his prejudices to so seriously warp his better judgment. Elated at his misconceived measurement of the side of the fundamental square base of the pyramid, Mr. Petrie considers he has ruined at the foundation the whole theory of Professor Smyth, and, believing that he has established the tombic theory, ironically implies that all the loftier ideas of the John Taylor school must be buried in the sarcophagus. So far, however, from coming with Mr. Petrie, and his too easily persuaded American friend, to a gloomy funeral, the modern builders of the pyramid theory may truly rejoice, and shout for joy, at the fitting way in which they now may crown the edifice, and cry, Grace unto its capstone! Berwick, Smyth, and Wood have completely vindicated, from Mr. Petrie's own labors, the 365.242 cubit length of the horizontal base side at the level of the lowest socket. Little, therefore, remains to be accomplished in this direction. So far, however, from weakening in other directions the firm conviction of all earnest students of this monument, that it was built harmoniously from block to block, and from base line to vertex, upon some consummate mathematical and cosmic scheme, Mr. Petrie's whole work, and many new ratios and proportions yet to be studied, and first discovered by himself, lead us to even firmer conviction that the monument is yet to assume proportions far beyond the most sanguine anticipations of those who look upon it as "a sign and wonder to the Lord of hosts." The particular dimensions of the elevated base line — that is, of the square marked out by the cutting of the monument through its own pavement level, and which Mr. Petrie makes to be around 9,068.8 inches — have noticeable π relations to the coffer far within, and to certain standard specific-gravity teachings of great import. So, too, the peculiar proportions found by Mr. Petrie to exist between the areas and elevations of every horizontal course of masonry throughout the monument, and the area of its meridian section, so as to bring out twenty-fifths (the all-important *cubit number*) thereof, each marked by specially thick courses, is too stupendous a piece of engineering, and too beautifully realized, not to have been based upon some grand fact which the architect felt that no undertaking would be too difficult to employ for monumentalizing.

But all of these, and many other similar studies, we leave for others to undertake, and shall ask the attention of our readers to a short study of the coffer dimensions, as viewed from Mr. Petrie's late measurements.

In "Our Inheritance in the Great Pyramid," Professor Smyth gives a table, in which he enumerates the actual measurements of the coffer, made on the spot by no less than twenty-five different observers, who, at different times during three full centuries, had preceded him. It is a remarkable fact, that they all differ, and some of them very materially. But three sets of these measurements are of any intrinsic scientific value, — those of Professor Greaves (1638), Jomard and Napoleon's French expedition (1799), and of Col. Vyse (1837). But, strange to say, even in these sets, several of the individual dimensions are so conclusively erratic, and at variance with the general average, as to warrant their summary rejection.

The measurements of Professor Smyth and Mr. Petrie are undoubtedly the most reliable ones that have yet been made, and perhaps are now the best that we can ever hope for from *direct* measurement, owing to the injured condition of this interesting relic, which is rapidly being mutilated past measurement. Centuries of pilgrims and professional iconoclasts cannot hammer at even the hardest granite without wearing it away at last. Thanks, however, to the Promethean proportions of the pyramid itself, and the carefully conceived harmonies of ratios running through all of its details, we can rest assured, that, from indirect measurements, — checks upon our work gathered from all parts of the pyramid, — we may yet arrive at the absolute *theoretical* size, shape, and proportions of this standard measure of capacity.

In the following table we subjoin the three several measures of Greaves, Jomard, and Vyse, considered as the most reliable by Professor Smyth, together with his own and those of Mr. Petrie. We have put the erratic dimensions in smaller type, as a most cursory examination will show they are of very little value.

COFFER MEASUREMENTS.

Designation.		Greaves. 1638.	Jomard. 1799.	Vyse. 1847.	Smyth. 1865.	Petrie. 1880-2.	Average. 1884.
Inside	Length	77.856	77.836	78.0	77.93	78.06	77.936
	Breadth	26.616	26.694	26.5	26.73	26.81	26.669
	Depth	34.320	37.285	34.5	34.34	34.42	34.395
Outside	Length	87.5	90.592	90.5	89.71	89.62	89.66
	Width	39.75	39.45	39.0	38.65	38.50	38.57
	Height	39.75	44.765	41.0	41.27	41.31	41.29
Thickness	Sides				5.99	5.82	5.905
	Bottom				6.92	6.89	6.905

From the several dimensions thus tabulated, we are enabled to calculate the corresponding volumes, as follows:—

		Brit. cu. in.
Greaves	Inside measurement	71,118
Vyse	Inside measurement	71,312
Smyth	Inside measurement	71,533
	Outside measurement	143,154
Petrie	Inside measurement	72,034
	Outside measurement	142,535
	Total	571,686
	Mean = $\frac{1}{8}$ =	71,460

Again, taking the dimensions for the inside from the column marked average, we obtain—

	Br. cu. in.
A volume of	71,489
And for the outside volume one of	142,830
One-half of the latter gives us	71,415
The mean of this, and the average inside, is equal to	71,452
The mean of the volumes from Smyth's and Petrie's measures, taken alone, is	71,542

Now, the absolute volume represented by 71,250 pyramid inches, each of (1.001 ± British inches)³, is

$$(71250)(1.001 \pm)^3 = 71463.963821250 \text{ British cu. in.,}$$

a quantity remarkably close to the mean of the whole table, as given above, and also close enough to those which we obtained from the column of average dimensions involving those of Greaves, Jomard, and Vyse, the latter of whom is generally accurate only as to whole numbers.

	Br. cu. in.
The mean of Smyth's measures, taken alone, is . .	71,529
While that of Petrie's is	71,516

Nor should it be overlooked in this connection, that Petrie himself allows that a possible error of from 60 to 100 ± British cubic inches may affect his results as to the coffer's volume; so that 71,516 — 71,464 = 52 is well within the limits of his lowest estimate as to error.

Viewing the whole subject broadly, then, and convinced that there are other reasons, of an entirely independent nature, which lend their weight in favor of the special volume of 71,250 pyramid cubic inches, or 71,463.9 + British cubic inches, we cannot but feel, that, in the present dilapidated condition of the coffer, our modern measures are as accurate as they ever will be made, and that we must depend upon the checks furnished elsewhere throughout the monument for correcting them. In view, too, of such and kindred facts, with all of which Mr. Petrie and other opponents of the metrological theory show themselves to be perfectly familiar, it is difficult to resist the effort to try and harmonize the details of the monument, not only with themselves, but with those grand cosmic truths to which they certainly approximate. And it is astonishing that scientific gentlemen, who, in other directions, strain themselves so desperately in their ceaseless search for "missing links" of a character absolutely degrading to intellectual man, will so allow their prejudices to close the eyes and ears of their understanding against the reception of images that perhaps may prove their origin to have rather been divine.

[Note IV.]

POSTSCRIPT.

The present volume does not pretend to be an answer to, or concern itself at all with, the style of arguments (?) advanced by Mr. Proctor and President Barnard. To ridicule is not to argue: to call design accident and coincidence, or else to deny its very identity as an intended realization of a means towards an end, merely because the preconceived opinions or teachings ot so-called science are opposed to it, is illogical and illiberal. Unless we are certain that modern science already stands upon the broad basis of absolute knowledge, it is ridiculous to shut out independent study, or to flatter ourselves that a monument such as that at Gizeh is not a harmony of cosmic truths. If the Great Pyramid, as we are now beginning to understand it, is, though full of sympathy with the diverse laws of nature and her ratios and proportions, still merely a tomb and a haphazard freak of chance, then, of all things upon earth, it is still, and ever will be, the most truly wonderful, and more than ever worthy of its place in history.

But there is a limit where the possibility of chance must end, and at which the convictions of even the most unwilling mind will be forced to recognize design. This limit has certainly been reached in matters relating to the Great Pyramid; and it is rather now the duty of our scholars to devote their attention to its study and to the unravelling of the message its originators intended it should convey to coming generations, than that any longer they should persist in belittling such a topic.

The metrologic scheme discussed in the foregoing pages has undoubtedly connected not only the teachings of the pyramid in one harmonious mosaic, but has joined them on to the study of nature, and of man himself. It has been entirely due to the lessons taught the writer by this monument, that any such scheme was even rendered conceivable; so that he is satisfied to leave it for those into whose hands it may come, to elaborate, until in some one science, which may be called metrology indeed, posterity may look from Nature up to Nature's God, and, glancing back again upon their neighbors, understand more fully how they are created in the image of their Maker.